NELSON SENIOR MATHS METHODS 12

FULLY WORKED SOLUTIONS

Chapter 4 Integration and areas

Exercise 4.01 The area under a curve

Concepts and techniques

1 **a** Distance travelled = $4 \times 80 = 320$ km

b Distance travelled = $7 \times 35 = 245$ km

c Distance travelled = $5 \times 50 = 250$ km

2 **a** Area = $3 \times 2 = 6$ units^2

b Area = $(5 - 2) \times 7 = 21$ units^2

3 **a** Approximate area = $10 \times 5 = 50$ units^2

b Approximate area = $12 \times 8 = 96$ units^2

c Approximate area = $(6 - 2) \times 25 = 100$ units^2

d Approximate area = $(3 - 1) \times 4 = 8$ units^2

e Approximate area = $(7 - 1) \times 6 = 36$ units^2

4 Approximate distance travelled = $8 \times 60 = 480$ km

5 **a** Area = $\frac{1}{2}(2+4)\times 6 = 18$ units^2

b Area = $\frac{1}{2}(25+10)\times 500 = 8750$ units^2

c Area = $\frac{1}{2}(8+4)\times 60 = 360$ units^2

d Area = $\frac{1}{2}(5+1)\times 30 = 90$ units^2

e Area = $\frac{1}{2}(80+40)\times 10 = 600$ units^2

6 Approximate area = $\frac{1}{2}(5-1)\times 4 = 8$ units^2

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Reasoning and communication

7 **a** Approximate area using triangle = $\frac{1}{2}(12 \times 30) = 180$ units^2

b Approximate area using trapezium = $\frac{1}{2}(5 + 30) \times 12 = 210$ units^2

8 **a** Area $\frac{1}{4}$ circle = $\frac{1}{4}(\pi r^2) = 6.25\pi$ (exactly) but ≈ 19.63 units^2

b **i** Approximate area = $4.5^2 = 20.25$ units^2

ii Approximate area = $\frac{1}{2}(5.5 \times 5.5) = 15.125$ units^2

9 **a** Volume $\approx 7 \times 300 = 2100$ kL

b Volume $\approx \frac{1}{2}(100 + 400) \times 7 = 1750$ kL

10 **a** Area $\frac{1}{2}$ circle = $\frac{1}{2}(\pi r^2) = 4.5\pi$ (exactly) but ≈ 14.14 unit^2

b **i** Area $\approx 3 \times 6 = 18$ units^2

ii Area $\approx \frac{1}{2}(6 \times 3) = 9$ units^2

iii Average area = $\frac{1}{2}(18 + 9) = 13.5$ units^2

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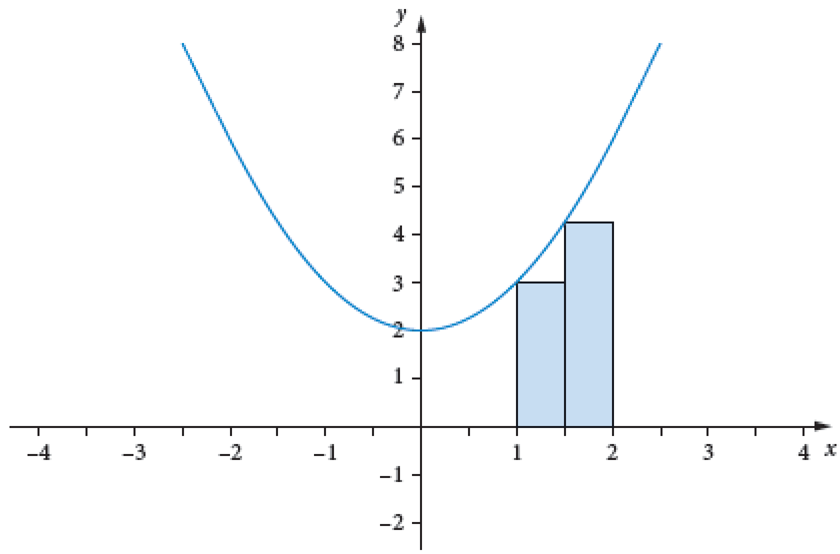
Exercise 4.02 Area approximations

Concepts and techniques

1 **a** The approximate area under the curve $y = x^2 + 2$ between $x = 1$ and $x = 2$

$= 0.5 \times f(1) + 0.5 \times f(1.5)$ where $f(x) = y$

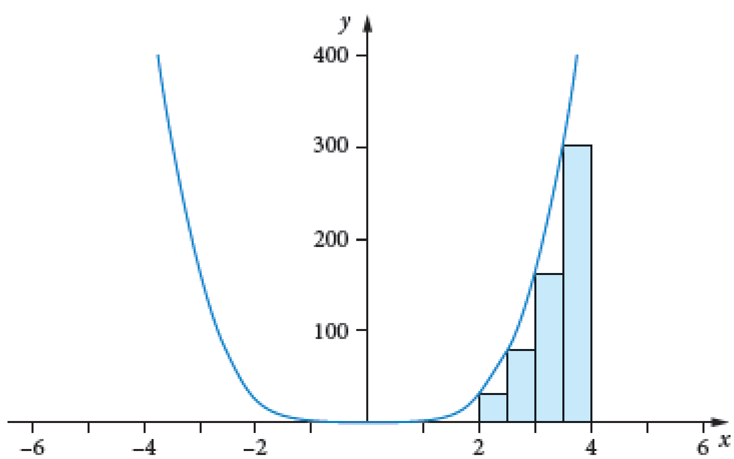


Approximate area $= 0.5 \times 3 + 0.5 \times 4.25$

$= 3.625$ units^2

b The approximate area under the curve $y = 2x^4$ between $x = 2$ and $x = 4$

$= 0.5 \times f(2) + 0.5 \times f(2.5) + 0.5 \times f(3) + 0.5 \times f(3.5)$ where $f(x) = y$



Approximate area $= 0.5 \times 32 + 0.5 \times 78.125 + 0.5 \times 162 + 0.5 \times 308.125$

$= 286.125$ units^2

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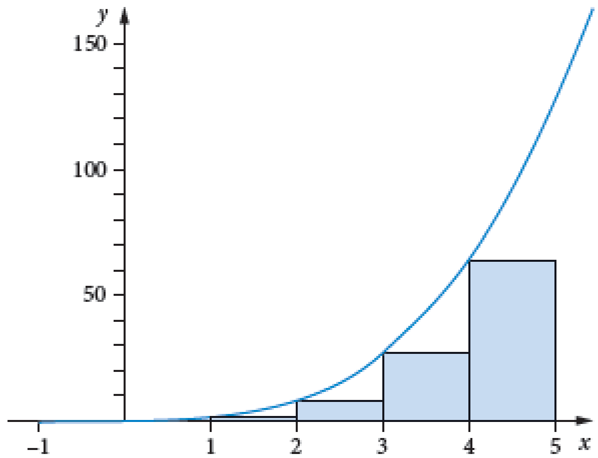
c The approximate area under the curve $y = x^3$ between $x = 1$ and $x = 5$

$= 1 \times f(1) + 1 \times f(2) + 1 \times f(3) + 1 \times f(4)$ where $f(x) = y$



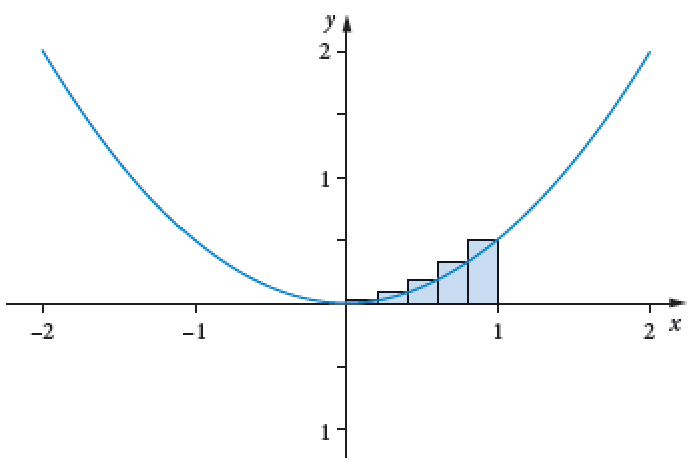
Approximate area $= 1 \times 1 + 1 \times 8 + 1 \times 27 + 1 \times 64$

$= 100$ units^2

d The approximate area under the curve $y = x^2$ between $x = 0$ and $x = 1$

$= 0.2 \times f(0.2) + 0.2 \times f(0.4) + 0.2 \times f(0.6) + 0.2 \times f(0.8) + 0.2 \times f(1)$

where $f(x) = y$



Approximate area $= 0.2 \times 0.04 + 0.2 \times 0.16 + 0.2 \times 0.36 + 0.2 \times 0.64 + 0.2 \times 1$

$= 0.44$ units^2

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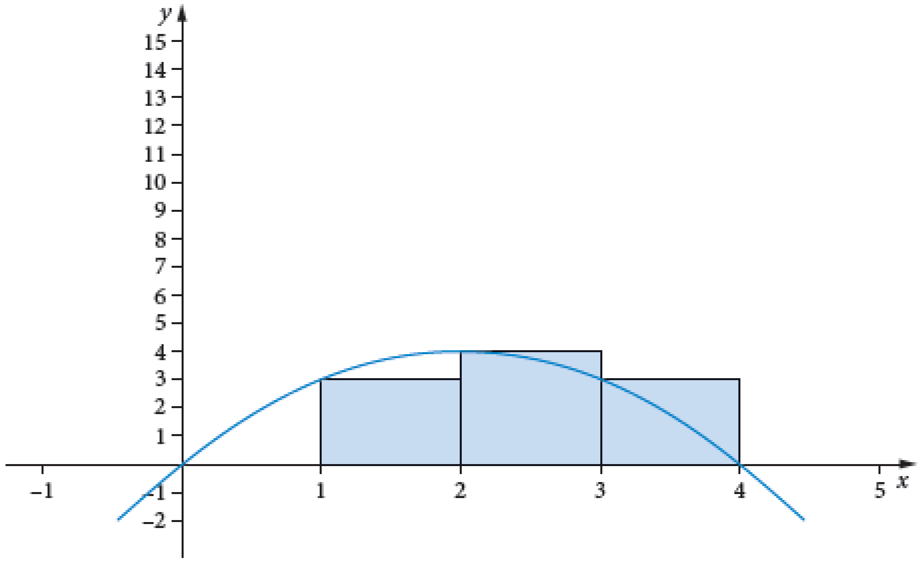
e The approximate area under the curve $y = 4x - x^2$ between $x = 1$ and $x = 4$

$= 1 \times f(1) + 1 \times f(2) + 1 \times f(3)$ where $f(x) = y$



Approximate area $= 1 \times 3 + 1 \times 4 + 1 \times 3$

$= 10$ units^2

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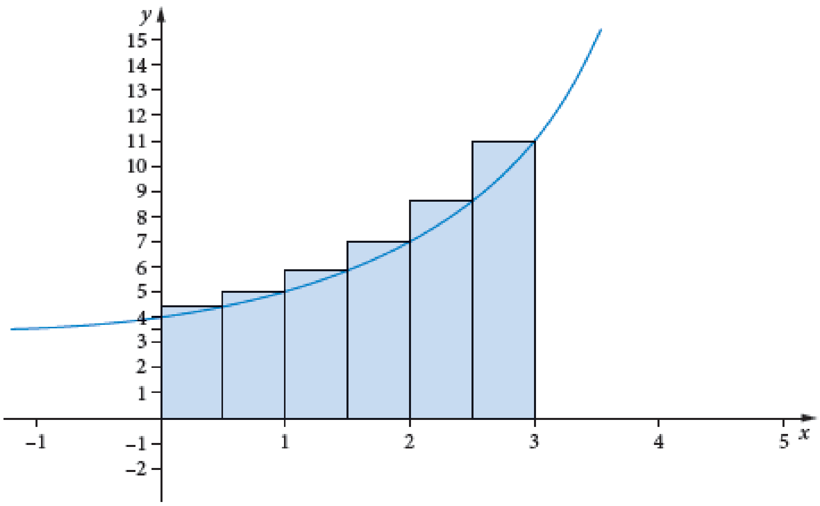
2 **a** The approximate area under the curve $y = 2^x + 3$ between $x = 0$ and $x = 3$

$= 0.5 \times f(0.5) + 0.5 \times f(1) + 0.5 \times f(1.5) + 0.5 \times f(2) + 0.5 \times f(2.5) + 0.5 \times f(3)$

$= 0.5[f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3)]$ where $f(x) = y$



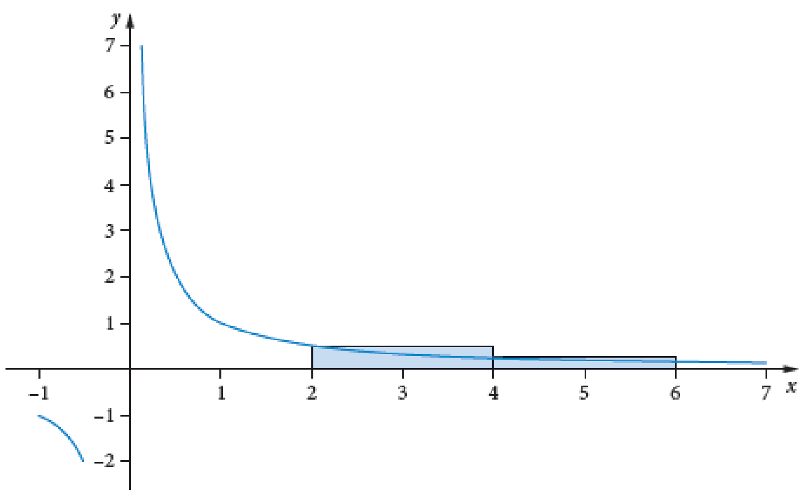
Approximate area $= 0.5[2^{0.5} + 3 + 2^1 + 3 + 2^{1.5} + 3 + 2^2 + 3 + 2^{2.5} + 3 + 2^3 + 3]$

$= 0.5[23.899 + 18]$

$= 20.95 \text{ units}^2$

b The approximate area under the curve $y = \dfrac{1}{x}$ between $x = 2$ and $x = 6$

$= 2 \times f(2) + 2 \times f(4)$ where $f(x) = y$

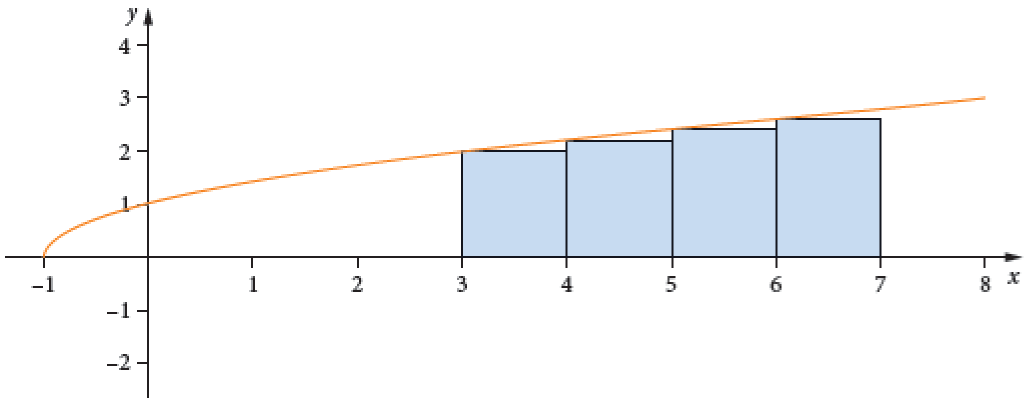


Approximate area $= 2 \times \dfrac{1}{2} + 2 \times \dfrac{1}{4}$

$= 1.5 \text{ units}^2$

c The approximate area under the curve $y = \sqrt{x+1}$ between $x = 3$ and $x = 7$

$= 1 \times f(3) + 1 \times f(4) + 1 \times f(5) + 1 \times f(6)$ where $f(x) = y$

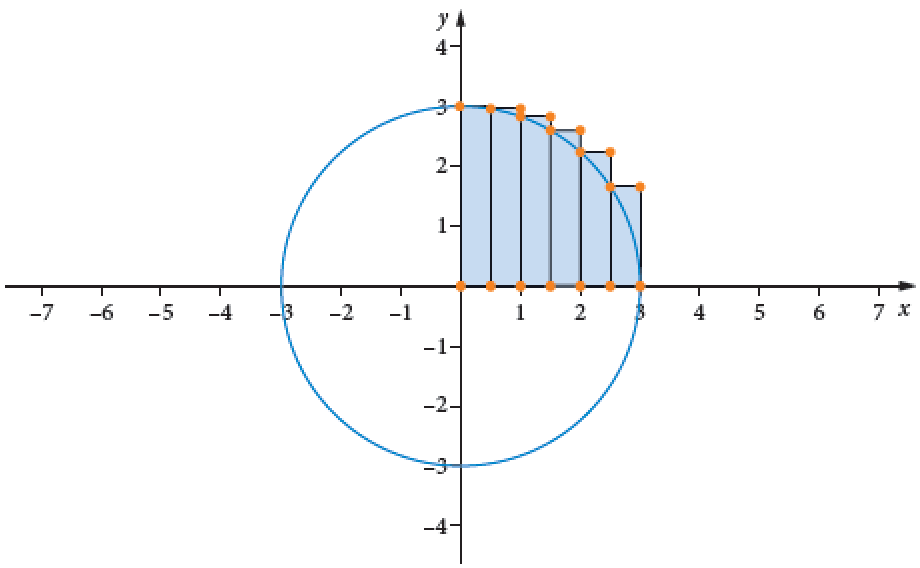


$$\text{Approximate area} = 2 + \sqrt{5} + \sqrt{6} + \sqrt{7}$$
$$= 9.33 \text{ units}^2$$

d The approximate area under the curve $y = \sqrt{9 - x^2}$ between $x = 0$ and $x = 3$

$= 0.5 \times f(0) + 0.5 \times f(0.5) + 0.5 \times f(1) + 0.5 \times f(1.5) + 0.5 \times f(2) + 0.5 \times f(2.5)$

$= 0.5[f(0) + f(0.5) + f(1) + f(1.5) + f(2) + f(2.5)\,]$ where $f(x) = y$



$$\text{Approximate area} = 0.5\left(\sqrt{9} + \sqrt{8.75} + \sqrt{8} + \sqrt{6.75} + \sqrt{5} + \sqrt{2.75}\right)$$
$$= 7.64 \text{ units}^2$$

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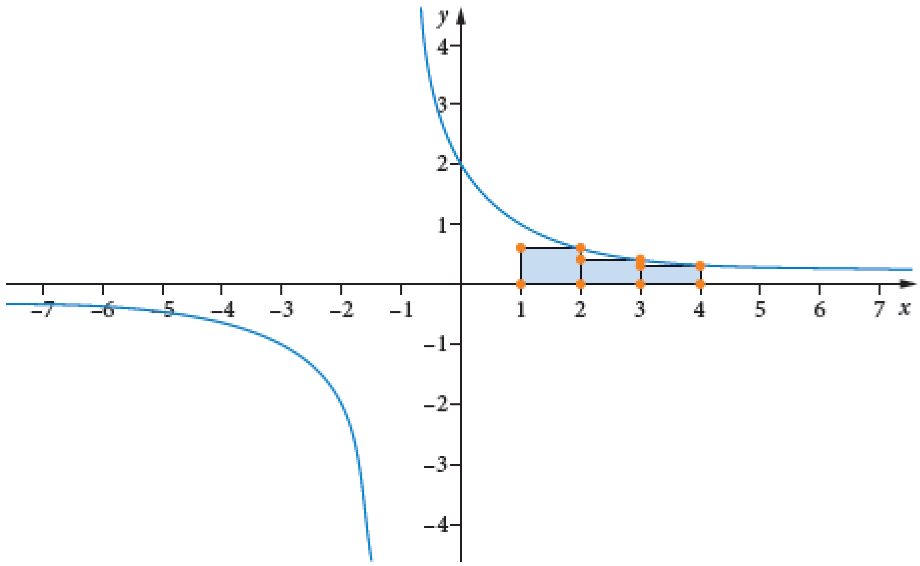
e The approximate area under the curve $y = \frac{2}{x+1}$ between $x = 1$ and $x = 4$

$= 1 \times f(2) + 1 \times f(3) + 1 \times f(4)$ where $f(x) = y$

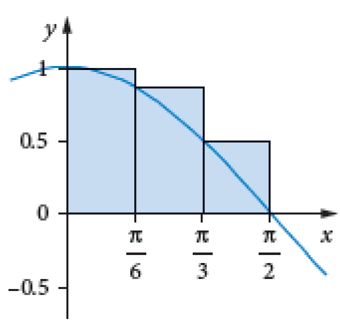


$$\text{Approximate area} = 1\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}\right)$$

$$= 1.57 \text{ units}^2$$

3 **a** The approximate area under the curve $y = \cos(x)$ between $x = 0$ and $x = \frac{\pi}{2}$

$$= \frac{\pi}{6} \times f(0) + \frac{\pi}{6} \times f(0.5) + \frac{\pi}{6} \times f(1) \text{ where } f(x) = y$$



$$\text{Approximate area} = 0.5[\cos(0) + \cos(0.5) + \cos(1)]$$

$$\approx 1.2388 \text{ units}^2$$

b Assume the domain is $0 < x < \frac{\pi}{2} \approx 1.570\,796$, $1.570\,796 \div 20 = 0.078\,5398$

Width = 0.078 5398 ≈ 0.08 units

Counter	x	$y = \cos x$
1	0	1
2	0.078 5398	0.996 91734
3	0.157 0796	0.987 688 35
4	0.235 6194	0.972 369 93
5	0.314 1592	0.951 056 54
6	0.392 699	0.923 879 56
7	0.471 2388	0.891 006 57
8	0.549 7786	0.852 640 22
9	0.628 3184	0.809 017 07
10	0.706 8582	0.760 406 06
11	0.785 398	0.707 1069
12	0.863 9378	0.649 448 19
13	0.942 4776	0.587 785 41
14	1.021 0174	0.522 498 75
15	1.099 5572	0.453 9907
16	1.178 097	0.382 683 66
17	1.256 6368	0.309 017 24
18	1.335 1766	0.233 445 63
19	1.413 7164	0.156 434 76
20	1.570 796	0.032 67
	sum =	**13.225 8523**

Area = 0.078 5398 × 13.225 8523 = 1.038 756 units^2

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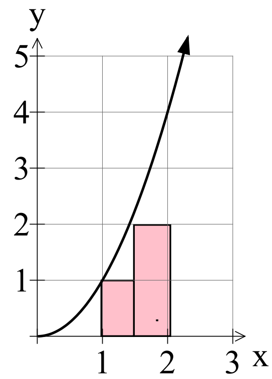
4 **a** **i** $y = x^2$ between $x = 1$ and $x = 2$ using two rectangles.

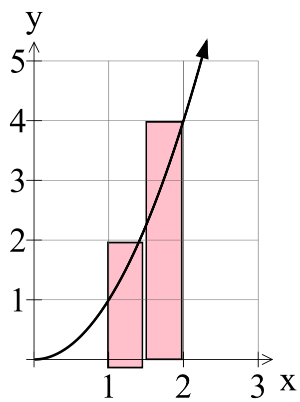


Approximate area $= 0.5 \times f(1) + 0.5 \times f(1.5)$ where $f(x) = y$

$= 0.5 + 1.125$

$= 1.625$ units^2

ii



Approximate area $= 0.5 \times f(1.5) + 0.5 \times f(2)$ where $f(x) = y$

$= 1.125 + 2$

$= 3.125$ units^2

iii 1 ÷ 20 = 0.05

Width = 0.05 units

Counter		$y = x^2$
1	1	1
2	1.05	1.1025
3	1.1	1.21
4	1.15	1.3225
5	1.2	1.44
6	1.25	1.5625
7	1.3	1.69
8	1.35	1.8225
9	1.4	1.96
10	1.45	2.1025
11	1.5	2.25
12	1.55	2.4025
13	1.6	2.56
14	1.65	2.7225
15	1.7	2.89
16	1.75	3.0625
17	1.8	3.24
18	1.85	3.4225
19	1.9	3.61
20	1.95	3.8025
	sum =	**45.175**

Area = 0.05 × 45.175 = 2.258 75 units^2

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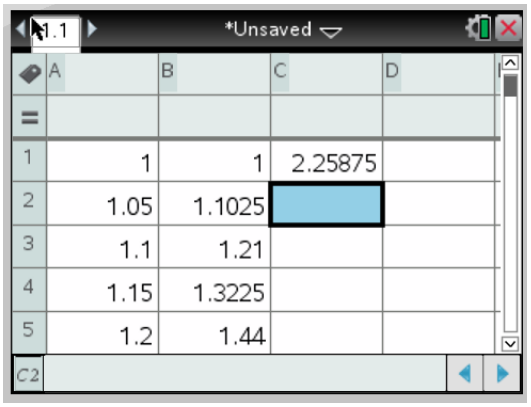

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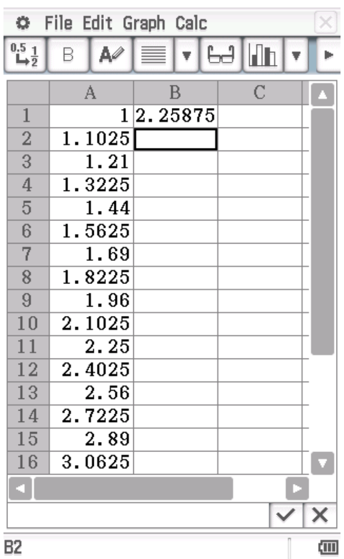
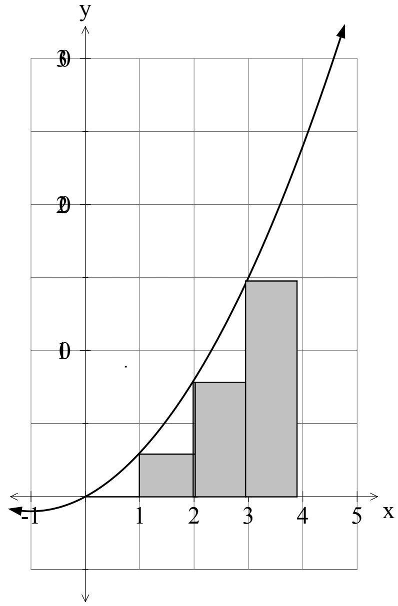


b $y = x^2 + 2x$ between $x = 0$ and $x = 4$ using four rectangles.

i Approximate area $= 1 \times f(0) + 1 \times f(1) + 1 \times f(2) + 1 \times f(3)$ where $f(x) = y$

$= 0 + 3 + 8 + 15$

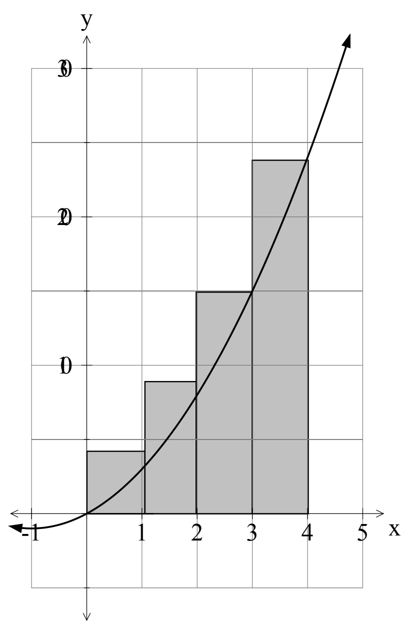
$= 26$ units^2



ii Approximate area $= 1 \times f(1) + 1 \times f(2) + 1 \times f(3) + 1 \times f(4)$ where $f(x) = y$

$= 3 + 8 + 15 + 24$

$= 50$ units^2



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iii $4 \div 20 = 0.2$

Width = 0.2

Counter		$y = x^2 + 2x$
1	0	0
2	0.2	0.44
3	0.4	0.96
4	0.6	1.56
5	0.8	2.24
6	1	3
7	1.2	3.84
8	1.4	4.76
9	1.6	5.76
10	1.8	6.84
11	2	8
12	2.2	9.24
13	2.4	10.56
14	2.6	11.96
15	2.8	13.44
16	3	15
17	3.2	16.64
18	3.4	18.36
19	3.6	20.16
20	3.8	22.04
	sum =	**174.8**

Area = $0.2 \times 174.8 = 34.96$ units^2

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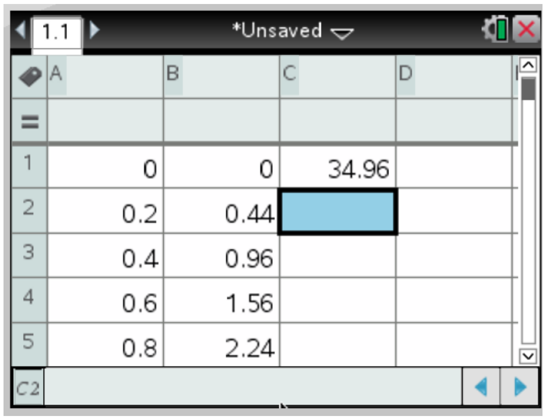

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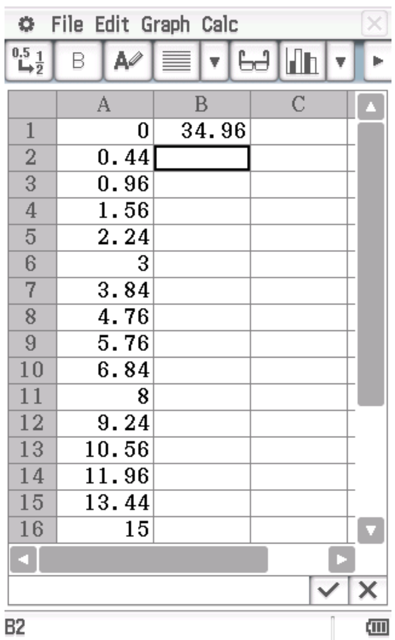

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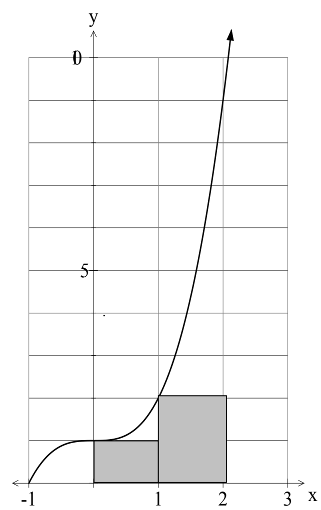
c $y = x^3 + 1$ between $x = 0$ and $x = 2$ using two rectangles.

i Approximate area $= 1 \times f(0) + 1 \times f(1)$ where $f(x) = y$

$= 1 + 2$

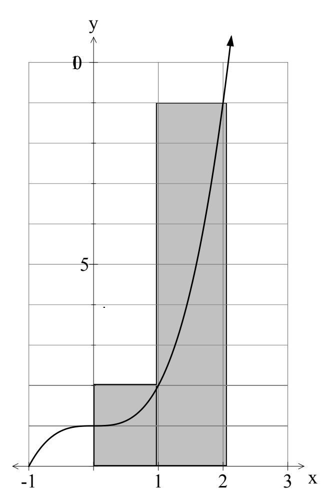
$= 3$ units^2



ii Approximate area $= 1 \times f(1) + 1 \times f(2)$ where $f(x) = y$

$= 2 + 9$

$= 11$ units^2



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iii $12 \div 20 = 0.1$

Width = 0.1

Counter		$y = x^3 + 1$
1	0	1
2	0.1	1.001
3	0.2	1.008
4	0.3	1.027
5	0.4	1.064
6	0.5	1.125
7	0.6	1.216
8	0.7	1.343
9	0.8	1.512
10	0.9	1.729
11	1	2
12	1.1	2.331
13	1.2	2.728
14	1.3	3.197
15	1.4	3.744
16	1.5	4.375
17	1.6	5.096
18	1.7	5.913
19	1.8	6.832
20	1.9	7.859
	sum =	**56.1**

Area = $0.1 \times 56.1 = 5.61$ units^2

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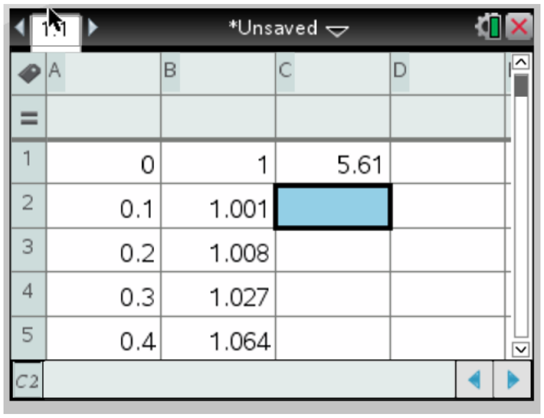

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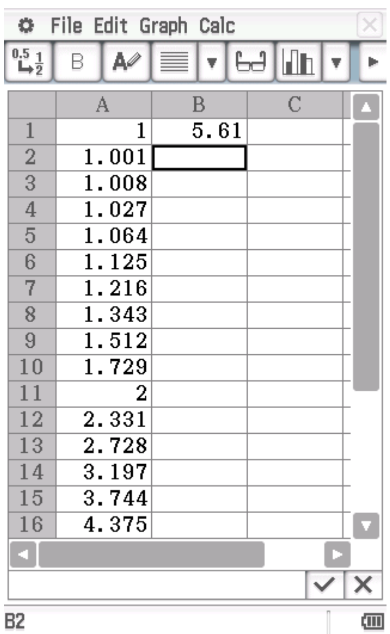

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d $y = x^2 - x - 2$ between $x = 2$ and $x = 4$ using four left rectangles.

i Approximate area $= \frac{1}{2} \times f(2) + \frac{1}{2} \times f\left(2\frac{1}{2}\right) + \frac{1}{2} \times f(3) + \frac{1}{2} \times f\left(3\frac{1}{2}\right)$

where $f(x) = x^2 - x - 2$

$= 0 + 0.875 + 2 + 3.375$

$= 6.25$

ii $y = x^2 - x - 2$ between $x = 2$ and $x = 4$ using four right rectangles.

Approximate area $= \frac{1}{2} \times f\left(2\frac{1}{2}\right) + \frac{1}{2} \times f(3) + \frac{1}{2} \times f\left(3\frac{1}{2}\right) + \frac{1}{2} \times f(4)$

$= 0.875 + 2 + 3.375 + 5$

$= 11.25$ units^2

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iii 2 ÷ 20 = 0.1

Width = 0.1 units

Counter		$y = x^2 - x - 2$
1	2	0
2	2.1	0.31
3	2.2	0.64
4	2.3	0.99
5	2.4	1.36
6	2.5	1.75
7	2.6	2.16
8	2.7	2.59
9	2.8	3.04
10	2.9	3.51
11	3	4
12	3.1	4.51
13	3.2	5.04
14	3.3	5.59
15	3.4	6.16
16	3.5	6.75
17	3.6	7.36
18	3.7	7.99
19	3.8	8.64
20	3.9	9.31
	sum =	**81.7**

Area = 0.1 × 81.7 = 8.17 units^2

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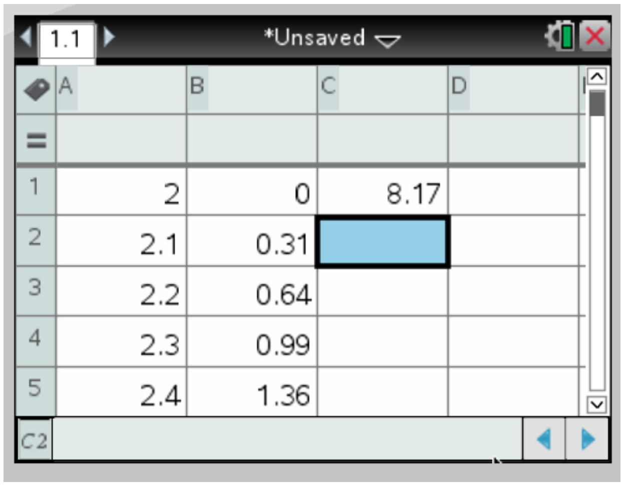

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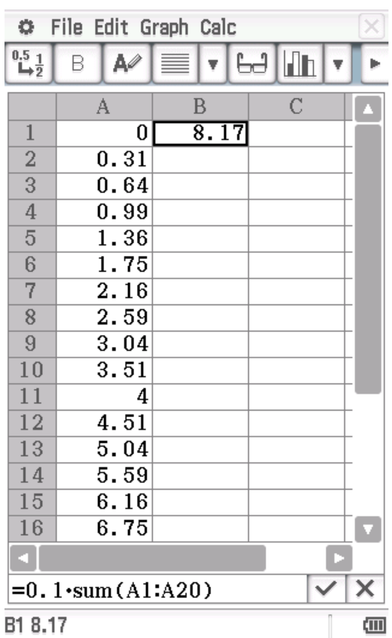

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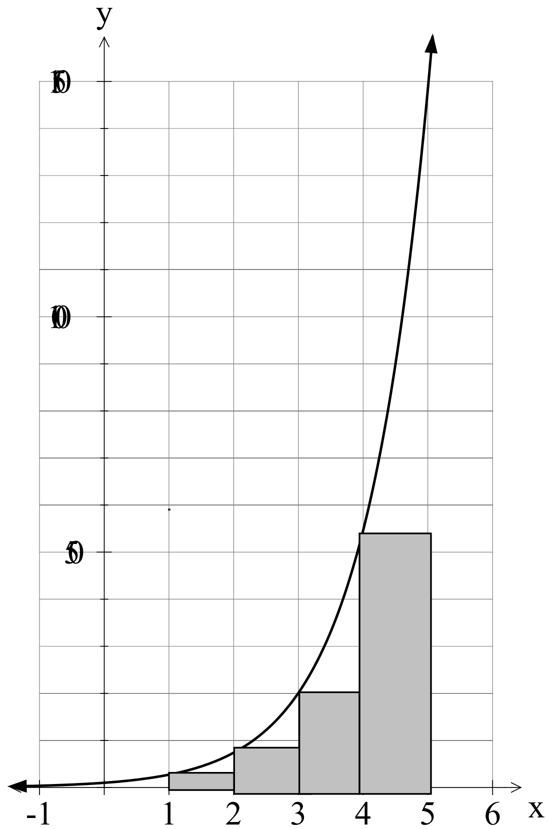
e $y = e^x$ between $x = 0$ and $x = 5$ using five rectangles.

i Approximate area $= 1 \times f(0) + 1 \times f(1) + 1 \times f(2) + 1 \times f(3) + 1 \times f(4)$

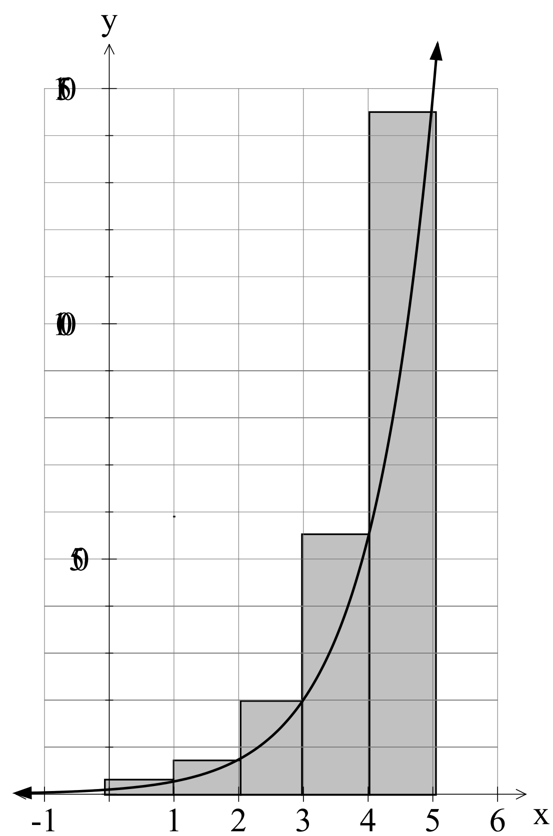
$= e^0 + e^1 + e^2 + e^3 + e^4$

$= 85.79$ units^2



ii $y = e^x$ between $x = 0$ and $x = 5$ using five rectangles.

$$\text{Approximate area} = 1 \times f(1) + 1 \times f(2) + 1 \times f(3) + 1 \times f(4) + 1 \times f(5)$$
$$= e^1 + e^2 + e^3 + e^4 + e^5$$
$$= 233.20 \text{ units}^2$$



iii $y = e^x$ between $x = 0$ and $x = 5$ using 20 left rectangles.

Width = 5 ÷ 20 = 0.25

Counter		$y = e^x$
1	0	1
2	0.25	1.284 025
3	0.5	1.648 721
4	0.75	2.117
5	1	2.718 282
6	1.25	3.490 343
7	1.5	4.481 689
8	1.75	5.754 603
9	2	7.389 056
10	2.25	9.487 736
11	2.5	12.182 49
12	2.75	15.642 63
13	3	20.085 54
14	3.25	25.790 34
15	3.5	33.115 45
16	3.75	42.521 08
17	4	54.598 15
18	4.25	70.105 41
19	4.5	90.017 13
20	4.75	115.5843
	sum =	**519.014**

Area = 0.25 × 519.04 = 129.75

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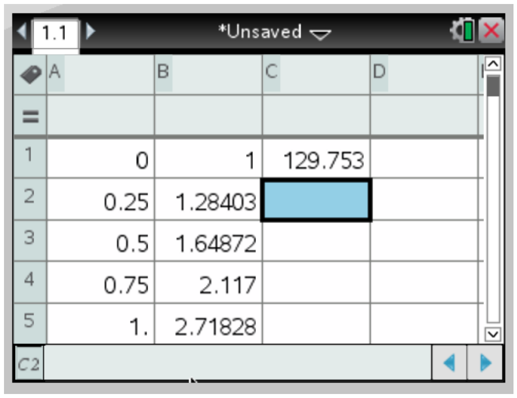

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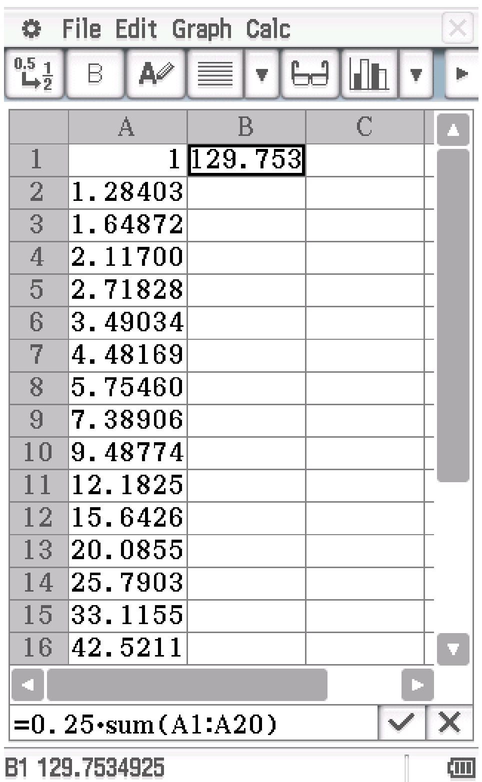

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5 **a** $y = x^2$ between $x = 1$ and $x = 2$ with 4 rectangles

$1 \div 4 = 0.25$

The right-value for the first rectangle is $1 + 0.25 = 1.25$, so its midpoint is 1.125.

Area $\approx 0.25 \times f(1.125) + 0.25 \times f(1.375) + 0.25 \times f(1.625) + 0.25 \times f(1.875)$

$= 0.25\ [f(1.125) + f(1.375) + f(1.625) + f(1.875)]$

$= 2.328$ units^2

b $y = x^3$ between $x = 0$ and $x = 1$ with 5 rectangles

$1 \div 5 = 0.2$

Area $\approx 0.2 \times f(0.1) + 0.2 \times f(0.3) + 0.2 \times f(0.5) + 0.2 \times f(0.7) + 0.2 \times f(0.9)$

$= 0.2\ [f(0.1) + f(0.3) + f(0.5) + f(0.7) + f(0.9)]$

$= 0.2446$ units^2

c $y = 2x^2 + 3$ between $x = 0$ and $x = 2$ with 4 rectangles

$2 \div 4 = 0.5$

Area $\approx 0.5 \times f(0.25) + 0.5 \times f(0.75) + 0.5 \times f(1.25) + 0.5 \times f(1.75)$

$= 0.5[\ f(0.25) + f(0.75) + f(1.25) + f(1.75)]$

$= 0.5[3.125 + 4.125 + 6.125 + 9.125]$

$= 1.25$ units^2

d $y = x^2 - 1$ between $x = 2$ and $x = 6$ with 8 rectangles

$4 \div 8 = 0.5$

Area $\approx 0.5 \times f(2.25) + 0.5 \times f(2.75) + 0.5 \times f(3.25) + \ldots + 0.5 \times f(5.75)$

$= 0.5\ [f\ (2.25) + f\ (2.75) + f\ (3.25) + \ldots.. + f\ (5.75)]$

$= 65.25$ units^2

e $y = \sin(x)$ between $x = 0$ and $x = 1$ with 10 rectangles

Width of interval $\Delta x = 1 \div 10 = 0.1$

Midpoints from $x = 0$ are 0.05, 0.15, ..., 0.95

Area $\approx 0.1 \times f(0.05) + 0.1 \times f(0.15) + 0.1 \times f(0.25) + \ldots + 0.1 \times f(0.95)$

$= 0.46$

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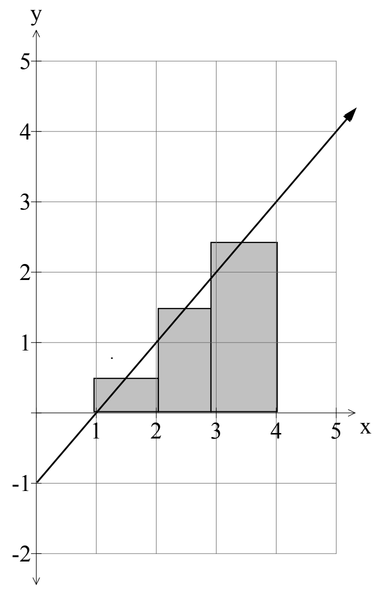
6 **a** Find the approximate area under the line $y = x - 1$ between $x = 1$ and $x = 4$ by using 3 centred rectangles.

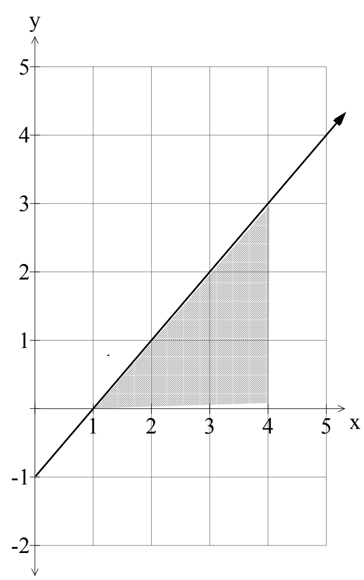


Area $\approx 1 \times f(1.5) + 1 \times f(2.5) + 1 \times f(3.5)$

$= 0.5 + 1.5 + 2.5$

$= 4.5$ units^2

b Using geometry



Area of the triangle $= 0.5 \times 3 \times 3$

$= 4.5$ units^2

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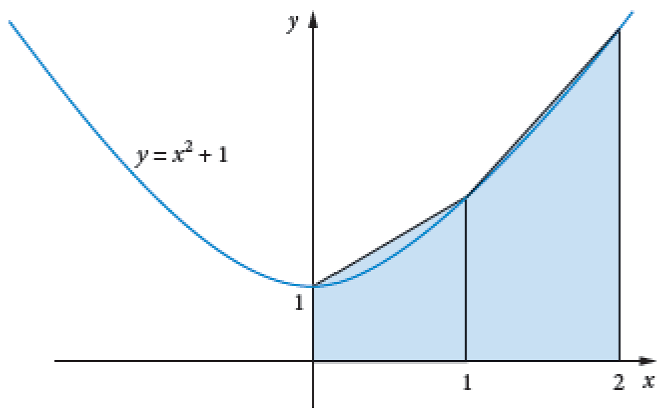
Reasoning and communication

7 Find an approximation to the area under the curve $y = x^2 + 1$ between $x = 0$ and $x = 2$ by using the sum of each trapezium.



Area = $T_1 + T_2$

$$= \frac{1}{2}\left[f(0)+f(1)\right]\times 1+\frac{1}{2}\left[f(1)+f(2)\right]\times 1$$

$$= \frac{1}{2}(1+2)\times 1+\frac{1}{2}(2+5)\times 1$$

$$= 5 \text{ units}^2$$

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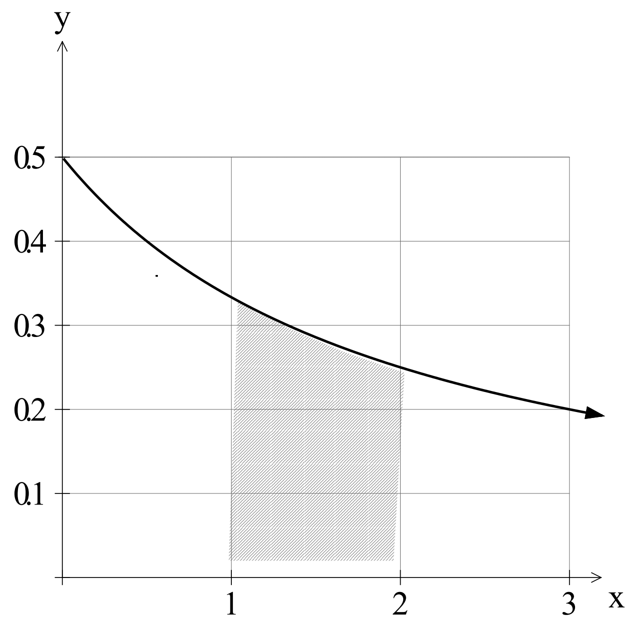
8 **a** Find the approximate area under the curve $y = \dfrac{1}{x+2}$ between $x = 1$ and $x = 2$.



i 4 left rectangles, $\Delta x = 0.25$. Use points 1, 1.25, 1.5, 1.75

Area $\approx 0.25[f(1) + f(1.25) + f(1.5) + f(1.75)]$

$= 0.298$ units^2

ii 4 right rectangles, $\Delta x = 0.25$. Use points 1.25, 1.5, 1.75, 2

Area $\approx 0.25[f(1.25) + f(1.5) + f(1.75) + f(2)]$

$= 0.278$ units^2

iii 4 centred rectangles, $\Delta x = 0.25$. Use points 1.125, 1.375, 1.625, 1.875

Area $\approx 0.25[f(1.125) + f(1.375) + f(1.625) + f(1.875)]$

$= 0.288$ units^2

b Area trapezium $= \dfrac{1}{2}\left[f(1) + f(2)\right] \times 1$

$= \dfrac{1}{2}\left(\dfrac{1}{3} + \dfrac{1}{4}\right) \times 1$

$= \dfrac{7}{24} = 0.292$ units^2

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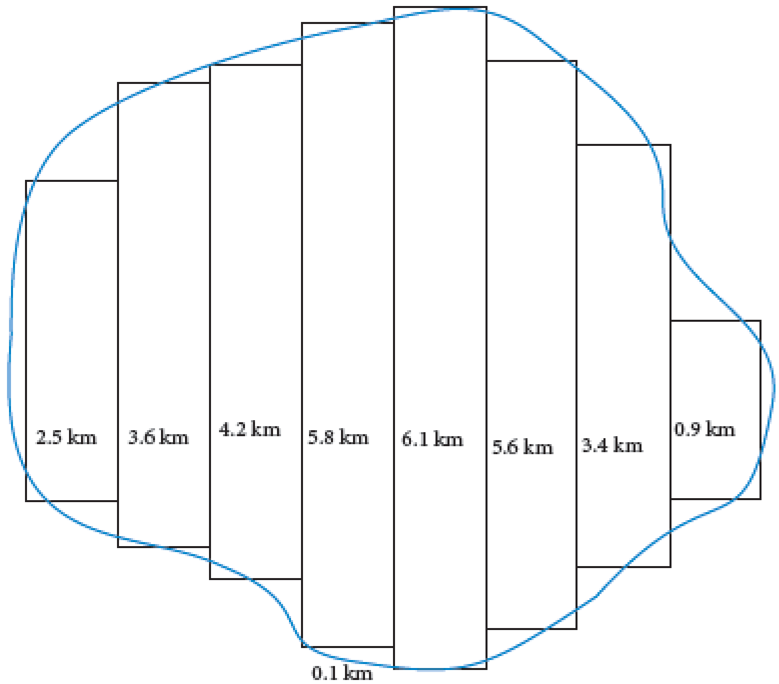
c Find the area using 50 centred rectangles. $\Delta x = 0.02$

Counter		$y=\dfrac{1}{x+2}$
1	1.01	0.332 226
2	1.03	0.330 033
3	1.05	0.327 869
4	1.07	0.325 733
5	1.09	0.323 625
6	1.11	0.321 543
7	1.13	0.319 489
8	1.15	0.317 46
9	1.17	0.315 457
10	1.19	0.313 48
11	1.21	0.311 526
12	1.23	0.309 598
13	1.25	0.307 692
14	1.27	0.305 81
15	1.29	0.303 951
16	1.31	0.302 115
17	1.33	0.300 3
18	1.35	0.298 507
19	1.37	0.296 736
20	1.39	0.294 985
21	1.41	0.293 255
22	1.43	0.291 545
23	1.45	0.289 855
24	1.47	0.288 184
25	1.49	0.286 533
26	1.51	0.284 9
27	1.53	0.283 286
28	1.55	0.281 69
29	1.57	0.280 112
30	1.59	0.278 552
31	1.61	0.277 008
32	1.63	0.275 482
33	1.65	0.273 973
34	1.67	0.272 48
35	1.69	0.271 003
36	1.71	0.269 542
37	1.73	0.268 097
38	1.75	0.266 667
39	1.77	0.265 252
40	1.79	0.263 852
41	1.81	0.262 467
42	1.83	0.261 097
43	1.85	0.259 74
44	1.87	0.258 398
45	1.89	0.257 069
46	1.91	0.255 754
47	1.93	0.254 453
48	1.95	0.253 165
49	1.97	0.251 889
50	1.99	0.250 627
	sum =	**14.384 06**

Area = $0.02 \times 14.384\ 06 = 0.287\ 68$ units^2

9 A lake has an irregular surface as shown below and an average depth of 850 metres.



a Area $\approx 0.1 \times [2.5 + 3.6 + 4.2 + 5.8 + 6.1 + 5.6 + 3.4 + 0.9]$

$= 0.1 \times 32.1$

$= 3.21 \text{ km}^2$

b Volume = area of top × average depth

$= 3.21 \times 0.85$

$= 2.7285 \text{ km}^3$

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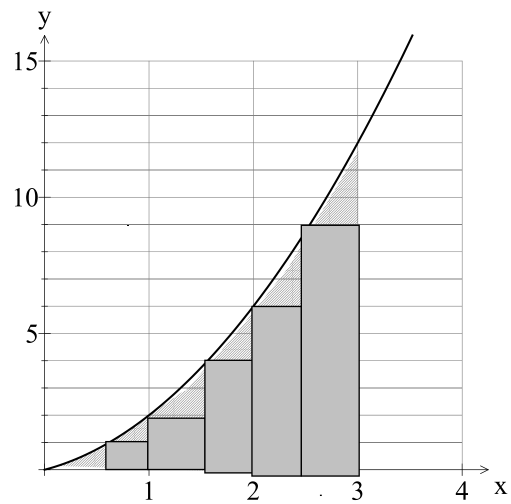
Exercise 4.03 The definite integral

Concepts and techniques

1 **a** $y = x^2 + x$ between $x = 0$ and $x = 3$ using 6 left rectangles.



$\Delta x = 0.5$. Use points 0, 0.5, 1, ..., 2.5

Area $\approx 0.5[f(0) + f(0.5) + f(1) + ... + f(2.5)]$

$= 10.625$ units^2

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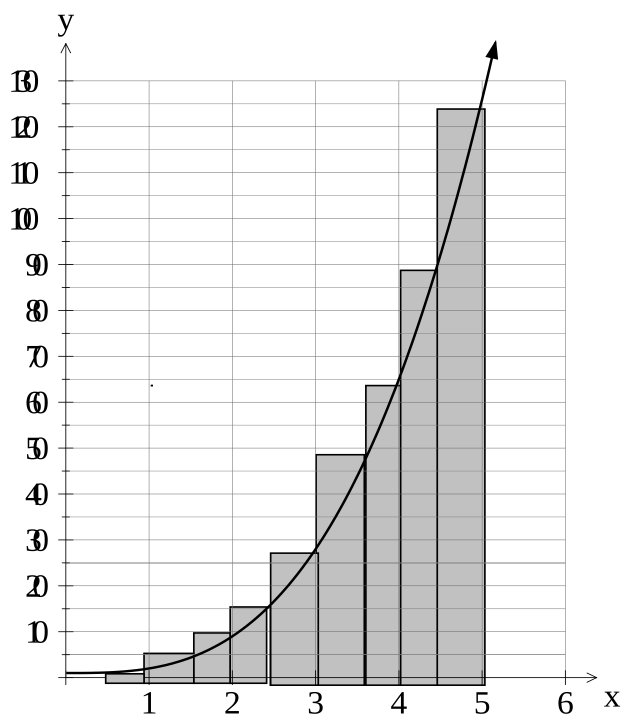
b $y = x^3 + 1$ between $x = 0$ and $x = 5$ using 10 right rectangles.



$\Delta x = 0.5$. Use points 0.5, 1, ..., 5

Area $\approx 0.5[f(0.5) + f(1) + ... + f(5)]$

$= 194.06$ units^2

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c $y = x^2 - 1$ between $x = 1$ and $x = 3$ using 8 left rectangles

$\Delta x = 0.25$. Use points 1, 1.25, 1.5, ..., 2.75

Area $\approx 0.25[f(1) + f(1.25) + ... + f(2.75)]$

$= 5.69$ units^2

d $y = x^4$ between $x = 0$ and $x = 6$ using 6 left rectangles

$\Delta x = 1$. Use points 0, 1, 2, ..., 5

Area $\approx 1 \times [f(0) + f(1) + f(2) + ... + f(5)]$

$= 979$ units^2

e $y = \sin(x)$ between $x = 0$ and $x = 3$ using 6 right rectangles

$\Delta x = 0.5$. Use points 0.5, 1, 1.5, 2, 2.5, 3

Area $\approx 0.5 \times [f(0.5) + f(1) + f(1.5) + ... + f(3)]$

$= 1.98$ units^2

2 **a** $\int_1^2 (x^2 + 2)dx$

$\Delta x = 0.125$

Use points 1.0625, 1.1875, 1.3125, 1.4375, 1.5625, 1.6875, 1.8125, 1.9375

Area $\approx 0.125 \times [f(0.0625) + f(0.1875) + f(0.3125) + ... + f(0.9375)]$

$= 4.33$ units^2

b $\int_2^4 2x^4 dx$

$\Delta x = 0.25$

Use points 2.125, 2.375, 2.625, 2.875, 3.125, 3.375, 3.625, 3.875

Area $\approx 0.125 \times [f(2.125) + f(2.375) + f(2.625) + ... + f(3.875)]$

$= 395.6$ units^2

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c $\int_1^5 x^3 dx$

$\Delta x = 0.5$

Use points 1.25, 1.75, 2.25, 2.75, 3.25, 3.75, 4.25, 4.75

Area ≈ 0.125 × [f (1.25) + f (1.75) + f (2.25) + ... + f (4.75)]

= 155.25 units^2

d $\int_3^7 \sqrt{x+1}\, dx$

$\Delta x = 0.5$

Use points 3.25, 3.75, 4.25, 4.75, 5.25, 5.75, 6.25, 6.75

Area ≈ 0.5 × [f (3.25) + f (3.75) + f (4.25) + ... + f (6.75)]

= 9.75 units^2

e $\int_1^9 (x^2 + 4x)dx$

$\Delta x = 1$. Use points 1.5, 2.5, 3.5, ..., 8.5

Area ≈ 0.5 × [f (1.5) + f (2.5) + f (3.5) + ... + f (8.5)]

= 402 units^2

3 **a** $\int_0^3 (2^x + 3)dx$

$\Delta x = 0.5$. Use points 0.5, 1, 1.5, ..., 2.5

Area ≈ 0.5 × [f (0.5) + f (1) + f (1.5) + ... + f (3)]

= 20.95 units^2

b $\int_2^5 \frac{1}{x} dx$

$\Delta x = 0.5$. Use points 2.5, 3, 3.5, ..., 5

Area ≈ 0.5 × [f (2.5) + f (3) + f (3.5) + ... + f (5)]

= 0.845 units^2

c $\int_0^3 \sqrt{9-x^2}\,dx$

$\Delta x = 0.5$. Use points 0.5, 1, 1.5, ..., 3

Area $\approx 0.5 \times [f(0.5) + f(1) + f(1.5) + ... + f(3)]$

$= 6.14$ units^2

d $\int_1^7 \frac{2}{x+1}\,dx$

$\Delta x = 1$. Use points 2, 3, 4, ..., 7

Area $\approx 1 \times [f(2) + f(3) + f(4) + ... + f(7)]$

$= 2.436$ units^2

e $\int_0^6 (x^3+2)\,dx$

$\Delta x = 1$. Use points 1, 2, 3, 4, 5, 6

Area $\approx 1 \times [f(1) + f(2) + f(3) + ... + f(6)]$

$= 453$ units^2

4 $\int_0^{\frac{\pi}{2}} \cos(x)\,dx$

$\Delta x = \frac{\pi}{4}$. Use points 0, $\frac{\pi}{4}$

Area $\approx \frac{\pi}{4} \times [f(0) + f(\frac{\pi}{4})]$

$= \frac{\pi}{4}\left[1+\frac{1}{\sqrt{2}}\right]$ units^2

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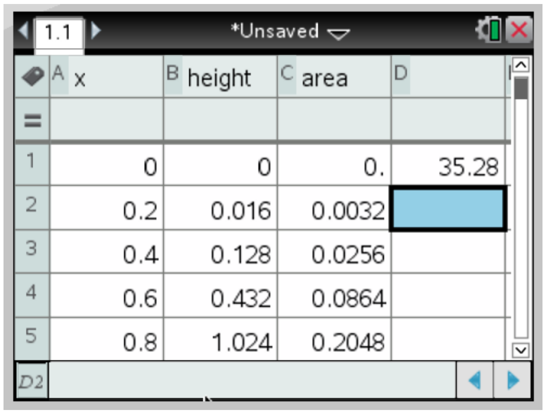
5 **a** $\int_0^3 2x^3 dx$, $\Delta x = \frac{3}{15} = 0.2$

width = 0.2

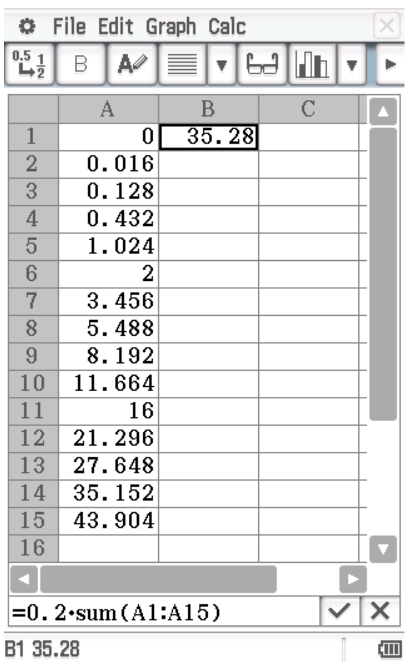
Counter		$y = 2x^3$
1	0	0
2	0.2	0.016
3	0.4	0.128
4	0.6	0.432
5	0.8	1.024
6	1	2
7	1.2	3.456
8	1.4	5.488
9	1.6	8.192
10	1.8	11.664
11	2	16
12	2.2	21.296
13	2.4	27.648
14	2.6	35.152
15	2.8	43.904
	sum =	**176.4**

Area = 0.2 × 176.4 = 35.28 units^2

TI-Nspire CAS



ClassPad



b $\int_1^4 (x^2+2)dx$, $\Delta x = \frac{3}{15} = 0.2$

width = 0.2

Counter		$y = x^2 + 2$
1	1	3
2	1.2	3.44
3	1.4	3.96
4	1.6	4.56
5	1.8	5.24
6	2	6
7	2.2	6.84
8	2.4	7.76
9	2.6	8.76
10	2.8	9.84
11	3	11
12	3.2	12.24
13	3.4	13.56
14	3.6	14.96
15	3.8	16.44
	sum =	**127.6**

Area = 0.2 × 127.6 = 25.52 units^2

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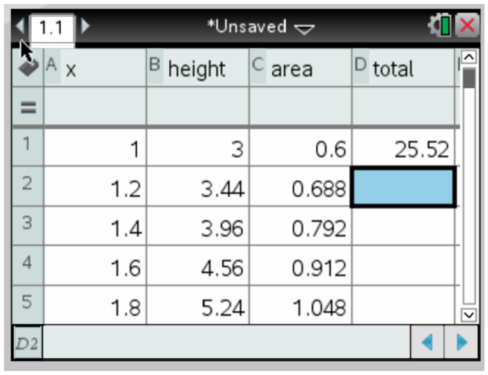
TI-Nspire CAS



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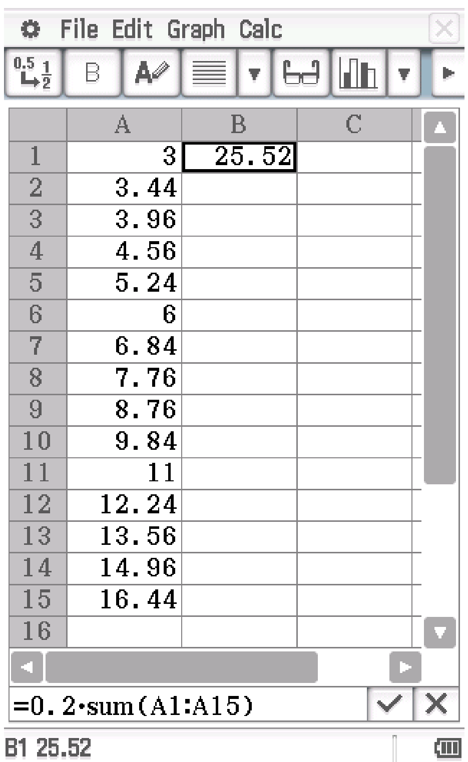

6 width = 0.5

Counter	x	$y = \frac{x-2}{x+1}$
1	2	0
2	2.5	0.142 86
3	3	0.25
4	3.5	0.333 33
5	4	0.4
6	4.5	0.454 55
7	5	0.5
8	5.5	0.538 46
9	6	0.571 43
10	6.5	0.6
5.11	7	0.625
12	7.5	0.647 06
13	8	0.666 67
14	8.5	0.684 21
15	9	0.7
16	9.5	0.714 29
17	10	0.727 27
18	10.5	0.739 13
19	11	0.75
20	11.5	0.76
	sum =	**10.80425**

Area = 0.5 × 10.804 25 = 5.402 units^2

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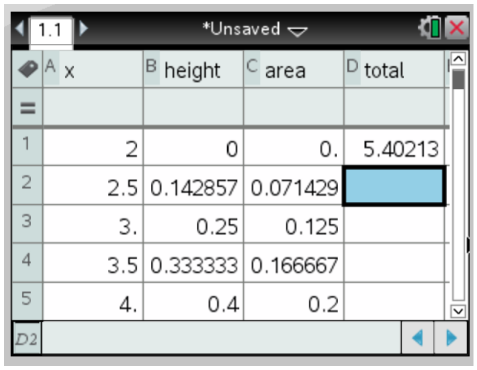
TI-Nspire CAS



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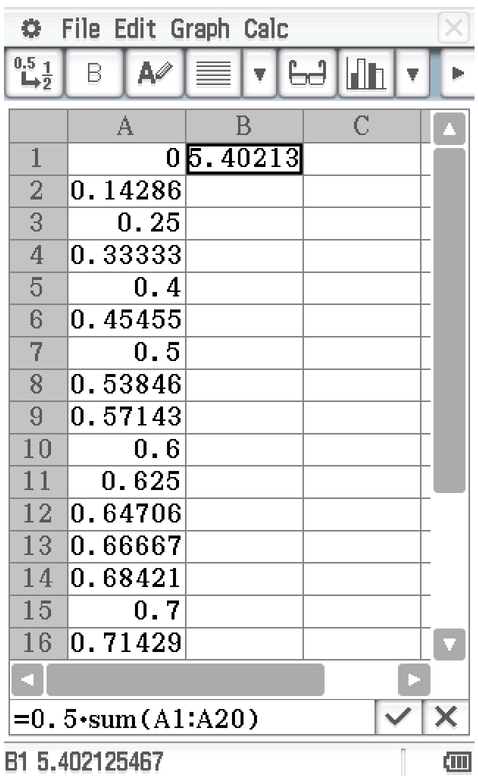

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7 $\int_1^6 (x^2 - 1)dx$, $\Delta x = \frac{5}{50} = 0.1$, so width = 0.1

Counter		$y = x^2 - 1$
1	1.1	0.21
2	1.2	0.44
3	1.3	0.69
4	1.4	0.96
5	1.5	1.25
6	1.6	1.56
7	1.7	1.89
8	1.8	2.24
9	1.9	2.61
10	2	3
11	2.1	3.41
12	2.2	3.84
13	2.3	4.29
14	2.4	4.76
15	2.5	5.25
16	2.6	5.76
17	2.7	6.29
18	2.8	6.84
19	2.9	7.41
20	3	8
21	3.1	8.61
22	3.2	9.24
23	3.3	9.89
24	3.4	10.56
25	3.5	11.25

Counter		$y = x^2 - 1$
26	3.6	11.96
27	3.7	12.69
28	3.8	13.44
29	3.9	14.21
30	4	15
31	4.1	15.81
32	4.2	16.64
33	4.3	17.49
34	4.4	18.36
35	4.5	19.25
36	4.6	20.16
37	4.7	21.09
38	4.8	22.04
39	4.9	23.01
40	5	24
41	5.1	25.01
42	5.2	26.04
43	5.3	27.09
44	5.4	28.16
45	5.5	29.25
46	5.6	30.36
47	5.7	31.49
48	5.8	32.64
49	5.9	33.81
50	6	35
	sum =	**684.25**

Area = 0.1 × 684.25 = 68.425 units^2

Reasoning and communication

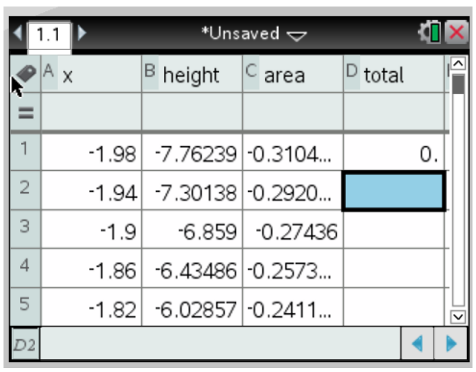
8 **a** Find an approximation to $\int_0^2 x^3 dx$ using 8 centred rectangles.

$\Delta x = 0.25$. Use points 0.125, 0.375, 0.625, ..., 1.875

Area $\approx 0.25 \times [f(0.125) + f(0.375) + f(0.625) + \ldots + f(1.875)]$

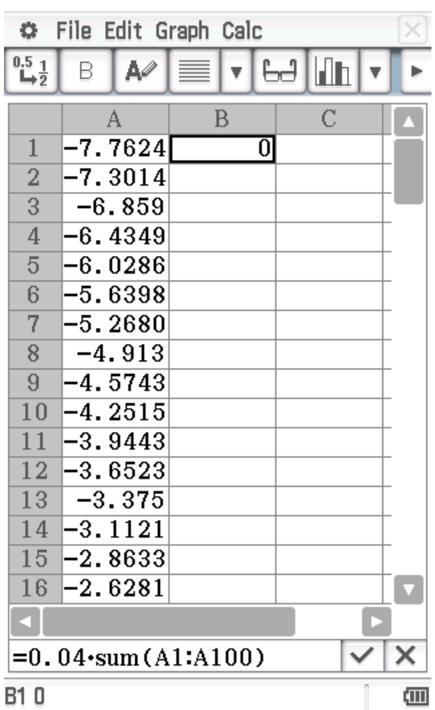
$= 3.97$ units^2

b TI-Nspire CAS



	A x	B height	C area	D total
=				
1	-1.98	-7.76239	-0.3104...	0.
2	-1.94	-7.30138	-0.2920...	
3	-1.9	-6.859	-0.27436	
4	-1.86	-6.43486	-0.2573...	
5	-1.82	-6.02857	-0.2411...	

ClassPad



	A	B	C
1	−7.7624	0	
2	−7.3014		
3	−6.859		
4	−6.4349		
5	−6.0286		
6	−5.6398		
7	−5.2680		
8	−4.913		
9	−4.5743		
10	−4.2515		
11	−3.9443		
12	−3.6523		
13	−3.375		
14	−3.1121		
15	−2.8633		
16	−2.6281		

=0.04•sum(A1:A100)

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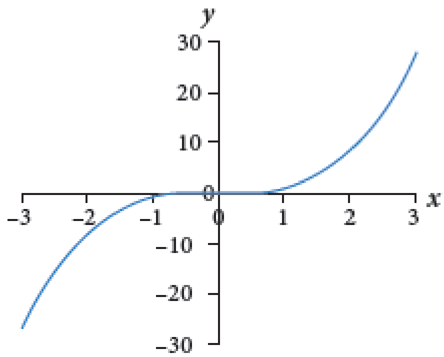
Find $\int_{-2}^{2} x^3 dx$ using 100 centred rectangles

$\Delta x = 0.04$. Use points –1.98, –1.94, –1.9, ..., 1.98

Area $\approx 0.04 \times [f(-1.98) + f(-1.94) + f(-1.9) + ... + f(1.98)]$

$= 0$ units^2

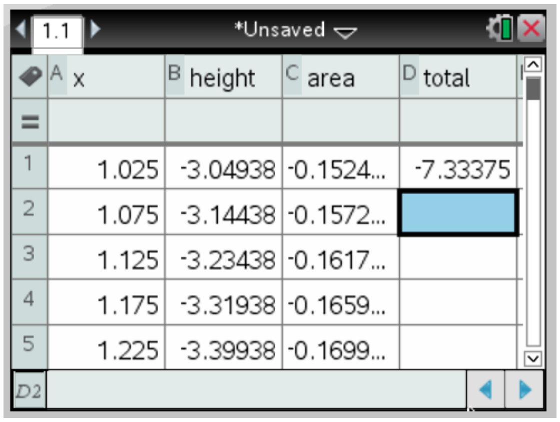
c Draw the graph of $y = x^3$ and explain the result in part **b**.



The 'area' as calculated using the y values for $-2 \le x \le 0$ is negative but has the same area as that for $0 < x < 2$ but this region has a positive value.

They cancel to give zero.

9 TI-Nspire CAS



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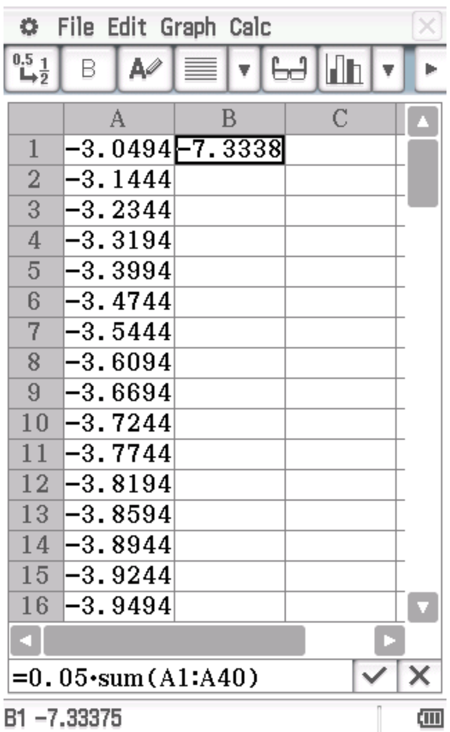


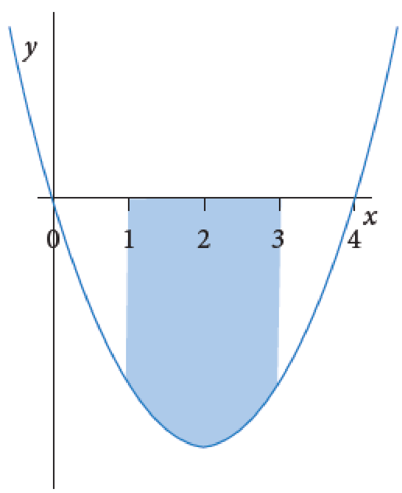
Evaluate $\int_1^3 (x^2 - 4x)dx$ using 40 centred rectangles.

$\Delta x = \dfrac{2}{40} = 0.05$, so width = 0.05

Counter		$y = x^2 - 4x$
1	1.025	–3.0494
2	1.075	–3.1444
3	1.125	–3.2344
4	1.175	–3.3194
5	1.225	–3.3994
6	1.275	–3.4744
7	1.325	–3.5444
8	1.375	–3.6094
9	1.425	–3.6694
10	1.475	–3.7244
11	1.525	–3.7744
12	1.575	–3.8194
13	1.625	–3.8594
14	1.675	–3.8944
15	1.725	–3.9244
16	1.775	–3.9494
17	1.825	–3.9694
18	1.875	–3.9844
19	1.925	–3.9944
20	1.975	–3.9994

Counter		$y = x^2 - 4x$
21	2.025	–3.9994
22	2.075	–3.9944
23	2.125	–3.9844
24	2.175	–3.9694
25	2.225	–3.9494
26	2.275	–3.9244
27	2.325	–3.8944
28	2.375	–3.8594
29	2.425	–3.8194
30	2.475	–3.7744
31	2.525	–3.7244
32	2.575	–3.6694
33	2.625	–3.6094
34	2.675	–3.5444
35	2.725	–3.4744
36	2.775	–3.3994
37	2.825	–3.3194
38	2.875	–3.2344
39	2.925	–3.1444
40	2.975	–3.0494
	sum =	**–146.675**

Area = 0.05 × (–146.675) = –7.333 75



The 'area' under the curve is all below the *x*-axis for $1 \le x \le 3$ so the answer to the calculation will be negative.

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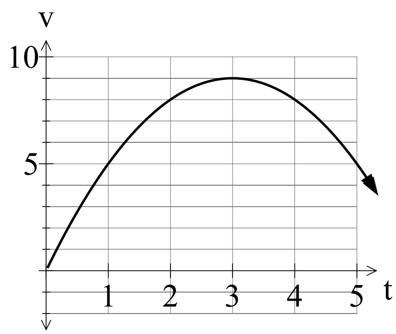
10 $v = 6t - t^2$ m/s and the initial position is at $x = 0$



a $\Delta x = 0.5$. Use points 0.25, 0.75, 1.25, ..., 3.75

Area $\approx 0.5 \times [f(0.25) + f(0.75) + f(1.25) + ... + f(3.75)]$

$= 26.75$ units^2

b $$\int_0^4 6t - t^2 \, dt = \left[3t^2 - \frac{t^3}{3}\right]_0^4$$

$$= \left(48 - \frac{64}{3}\right) - (0 - 0)$$

$$= 26\frac{2}{3}\text{ m}$$

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Exercise 4.04 Properties of the definite integral

Concepts and techniques

1 **a** **i** $\int_1^4 x^3 dx \approx 1 \times [f(1) + f(2) + f(3)]$

$= 1^3 + 2^3 + 3^3$

$= 36$

ii $\int_4^6 x^3 dx \approx 1 \times [f(4) + f(5)]$

$= 4^3 + 5^3$

$= 189$

iii $\int_1^6 x^3 dx \approx 1 \times [f(1) + f(2) + f(3) + f(4) + f(5)]$

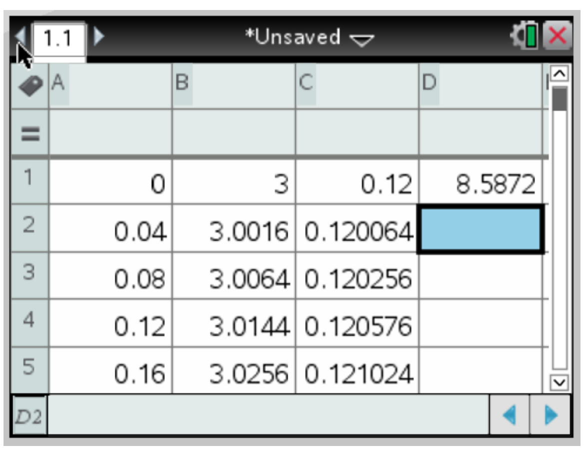
$= 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

$= 225$

b $\int_1^4 x^3 dx + \int_4^6 x^3 dx = 36 + 189 = 225 = \int_1^6 x^3 dx$

$\therefore \int_1^6 x^3 dx = \int_1^4 x^3 dx + \int_4^6 x^3 dx$

2 TI-Nspire CAS



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	A	B	C	D
1	2	7	0.28	24.4272
2	2.04	7.1616	0.286464	
3	2.08	7.3264	0.293056	
4	2.12	7.4944	0.299776	
5	2.16	7.6656	0.306624	

D2

	A	B	C	D
1	0	3	0.12	33.0144
2	0.04	3.0016	0.120064	
3	0.08	3.0064	0.120256	
4	0.12	3.0144	0.120576	
5	0.16	3.0256	0.121024	

D2

ClassPad

	A	B	C
1	3	8.5872	
2	3.0016		
3	3.0064		
4	3.0144		
5	3.0256		
6	3.04		
7	3.0576		
8	3.0784		
9	3.1024		
10	3.1296		
11	3.16		
12	3.1936		
13	3.2304		
14	3.2704		
15	3.3136		
16	3.36		

3

A1

	A	B	C
1	7	24.4272	
2	7.1616		
3	7.3264		
4	7.4944		
5	7.6656		
6	7.84		
7	8.0176		
8	8.1984		
9	8.3824		
10	8.5696		
11	8.76		
12	8.9536		
13	9.1504		
14	9.3504		
15	9.5536		
16	9.76		

7

A1 7

	A	B	C
1	3	33.0144	
2	3.0016		
3	3.0064		
4	3.0144		
5	3.0256		
6	3.04		
7	3.0576		
8	3.0784		
9	3.1024		
10	3.1296		
11	3.16		
12	3.1936		
13	3.2304		
14	3.2704		
15	3.3136		
16	3.36		

=0.04•sum(A1:A100)

B1 33.0144

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a **i** $\int_0^2 (x^2+3)dx \approx 8.58$

ii $\int_2^4 (x^2+3)dx \approx 24.43$

iii $\int_0^4 (x^2+3)dx \approx 33.01$

b $\int_0^2 (x^2+3)dx + \int_2^4 (x^2+3)dx = 8.58 + 24.43 = 33.01$

$\int_0^4 (x^2+3)dx \approx 33.01$

They are exactly the same.

3 **a** $\int_0^1 x^2 dx + \int_1^5 x^2 dx = \int_0^5 x^2 dx$

b $\int_1^4 (x+1)dx + \int_4^7 (x+1)dx = \int_1^7 (x+1)dx$

c $\int_{-2}^0 (x^3 - x - 1)dx + \int_0^2 (x^3 - x - 1)dx = \int_{-2}^2 (x^3 - x - 1)dx$

d $\int_0^2 (2x+1)dx + \int_2^3 (2x+1)dx = \int_0^3 (2x+1)dx$

e $\int_1^2 6x^3 dx + \int_2^3 6x^3 dx = \int_1^3 6x^3 dx$

f $\int_{-1}^1 (3x^2 - 4x - 1)dx + \int_1^3 (3x^2 - 4x - 1)dx = \int_{-1}^3 (3x^2 - 4x - 1)dx$

g $\int_{-2}^0 (x^2 - 2)dx + \int_0^2 (x^2 - 2)dx = \int_{-2}^2 (x^2 - 2)dx$

h $\int_0^3 3dx + \int_3^7 3dx = \int_0^7 3dx$

i $\int_1^2 5x^4 dx + \int_2^3 5x^4 dx = \int_1^3 5x^4 dx$

j $\int_0^4 (2x-3)dx + \int_4^6 (2x-3)dx = \int_0^6 (2x-3)dx$

4 **a** **i** $\int_0^{10} x^2 dx \approx 332.5$

ii $\int_0^{10} 3x^2 dx \approx 997.5$

b $3 \times \int_0^{10} x^2 dx \approx 3 \times 332.5 = 997.5 = \int_0^{10} 3x^2 dx$

$\therefore \int_0^{10} 3x^2 dx = 3\int_0^{10} x^2 dx$

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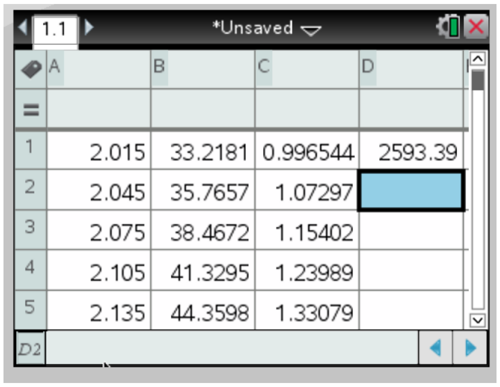
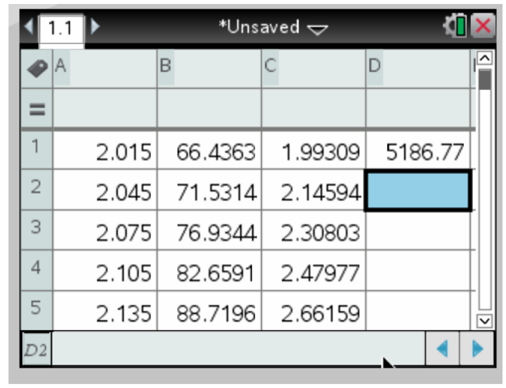
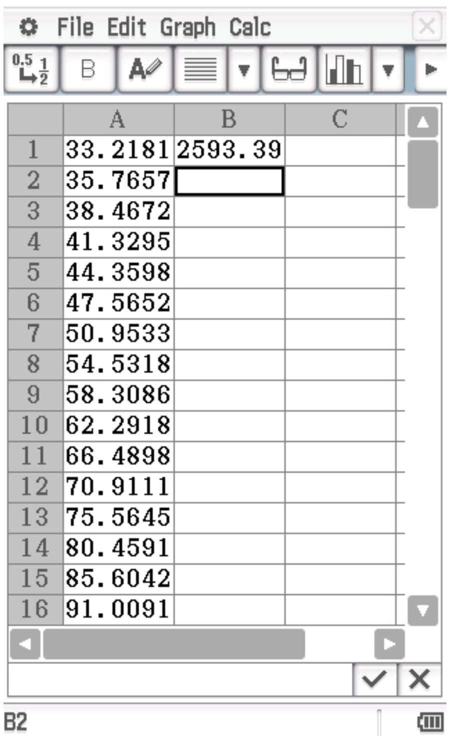
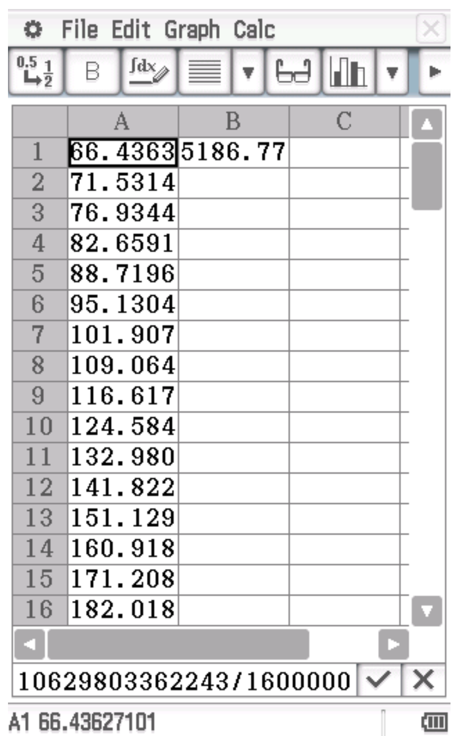
5 TI-Nspire CAS





ClassPad





a **i** $\int_2^5 x^5 dx \approx 2593.39$ using $\Delta x = 0.03$ and points 2.015, 2.045, etc., to 4.985

ii $\int_2^5 2x^5 dx \approx 5186.77$

b $2\int_2^5 x^5 dx = 2 \times 2593.39 = 5186.78 \approx \int_2^5 2x^5 dx$

$\therefore \int_2^5 2x^5 dx = 2\int_2^5 x^5 dx$

6 **a** **i** $\int_1^2 3x\,dx \approx 4.125$

ii $\int_1^2 2x^2 dx \approx 3.938$

iii $\int_1^2 (2x^2 + 3x)dx \approx 8.0625$

b $\int_1^2 2x^2 dx + \int_1^2 3x\,dx = 4.125 + 3.938 = 8.063 = \int_1^2 (2x^2 + 3x)dx$

$\therefore \quad \int_1^2 (2x^2 + 3x)dx = \int_1^2 2x^2 dx + \int_1^2 3x\,dx$

7 **a** $\int_0^2 (3x^2 + 2)dx + \int_0^2 2x\,dx = \int_0^2 (3x^2 + 2 + 2x)dx$

b $\int_1^2 x^3 dx + \int_1^2 (2x^3 - 3x + 1)dx = \int_1^2 (3x^3 - 3x + 1)dx$

c $\int_{-1}^1 (2x^4 + 3)dx + \int_{-1}^1 (x^3 - x^2 - 4)dx = \int_{-1}^1 (2x^4 + x^3 - x^2 - 1)dx$

d $\int_0^3 (x^2 + 4x - 3)dx + \int_0^3 (x^2 - x - 1)dx = \int_0^3 (2x^2 + 3x - 4)dx$

e $\int_1^5 2x\,dx + \int_1^5 7dx = \int_1^5 \left(2x + 7\right)dx$

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Reasoning and communication

8 **a** **i** $\int_2^6 x^3 dx \approx 270$

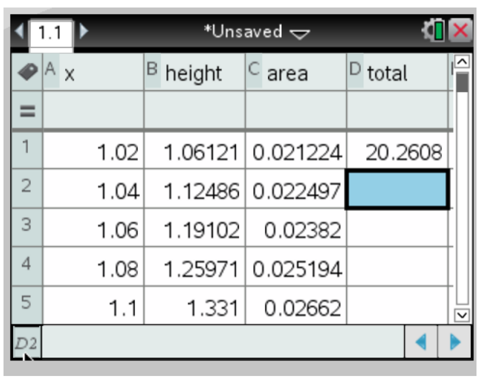
ii $\int_2^6 x^2 dx \approx 61.5$

iii $\int_2^6 (x^3 - x^2) dx \approx 208.5$

b $\int_2^6 x^3 dx - \int_2^6 x^2 dx = 270 - 61.5 = 208.5 = \int_2^6 (x^3 - x^2) dx$

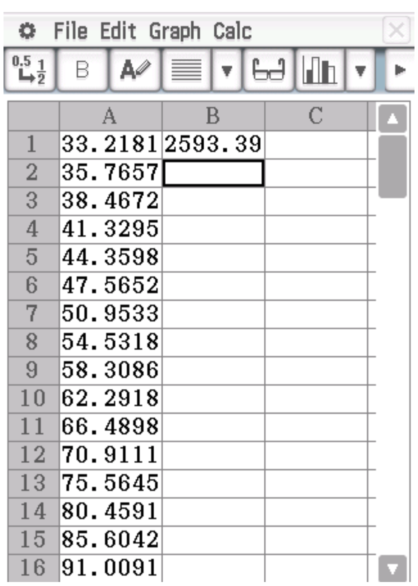
$\therefore \int_2^6 (x^3 - x^2) dx = \int_2^6 x^3 dx - \int_2^6 x^2 dx$

9 TI-Nspire CAS

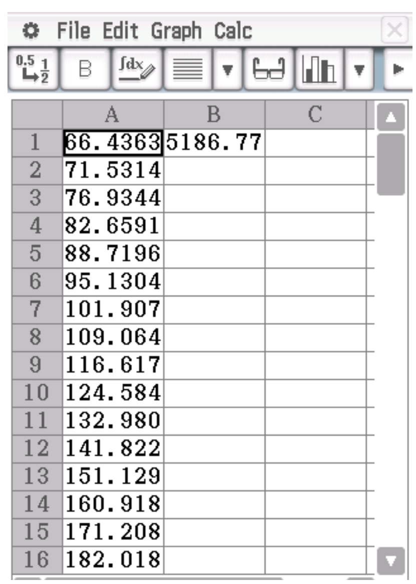


	A x	B height	C area	D total
1	1.02	1.06121	0.021224	20.2608
2	1.04	1.12486	0.022497	
3	1.06	1.19102	0.02382	
4	1.08	1.25971	0.025194	
5	1.1	1.331	0.02662	

ClassPad



	A	B	C
1	33.2181	2593.39	
2	35.7657		
3	38.4672		
4	41.3295		
5	44.3598		
6	47.5652		
7	50.9533		
8	54.5318		
9	58.3086		
10	62.2918		
11	66.4898		
12	70.9111		
13	75.5645		
14	80.4591		
15	85.6042		
16	91.0091		



	A	B	C
1	66.4363	5186.77	
2	71.5314		
3	76.9344		
4	82.6591		
5	88.7196		
6	95.1304		
7	101.907		
8	109.064		
9	116.617		
10	124.584		
11	132.980		
12	141.822		
13	151.129		
14	160.918		
15	171.208		
16	182.018		

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a $\int_1^3 x^3 dx \approx 20.26$

b $\int_3^1 x^3 dx = \left[\frac{x^4}{4}\right]_3^1 = \frac{1}{4}(1-81) = -20.26$

c $\int_1^3 x^3 dx = -\int_3^1 x^3 dx$

d $\int_a^b f(x)dx + \int_b^a f(x)dx = \int_a^a f(x)dx = 0$

$\therefore \int_a^b f(x)dx = -\int_b^a f(x)dx$

10 $v = 6t - t^2$ m/s

Distance travelled = $\int_0^2 6t - t^2 dt \approx 30.75$ m

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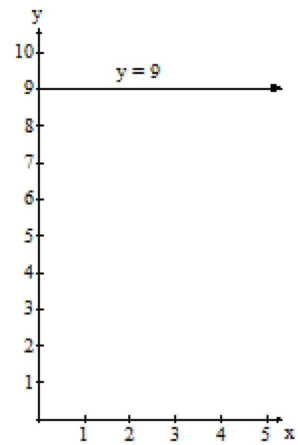
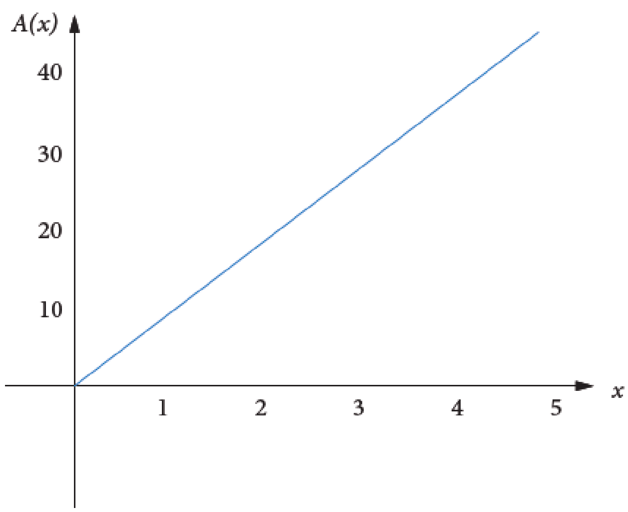
Exercise 4.05 The fundamental theorem of calculus

Concepts and techniques

1 **a**

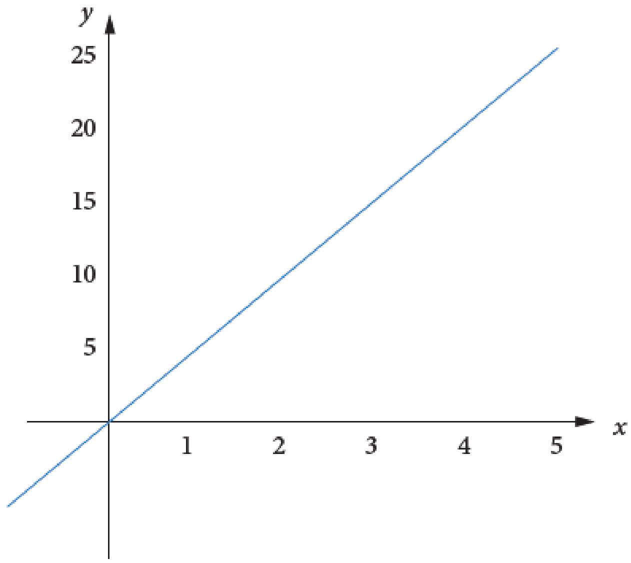


Interval	Area
[0, 0]	0
[0, 1]	9
[0, 2]	18
[0, 3]	27
[0, 4]	36



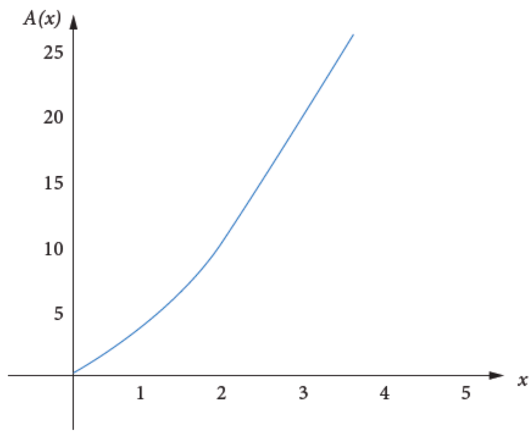
b $A(x) = 9x$

2 **a**



b

Interval	Area
[0, 0]	0
[0, 1]	3
[0, 2]	12
[0, 3]	27
[0, 4]	48



c $A(x) = 3x^2$

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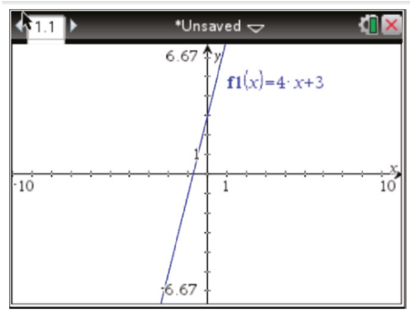
3 a TI-Nspire CAS



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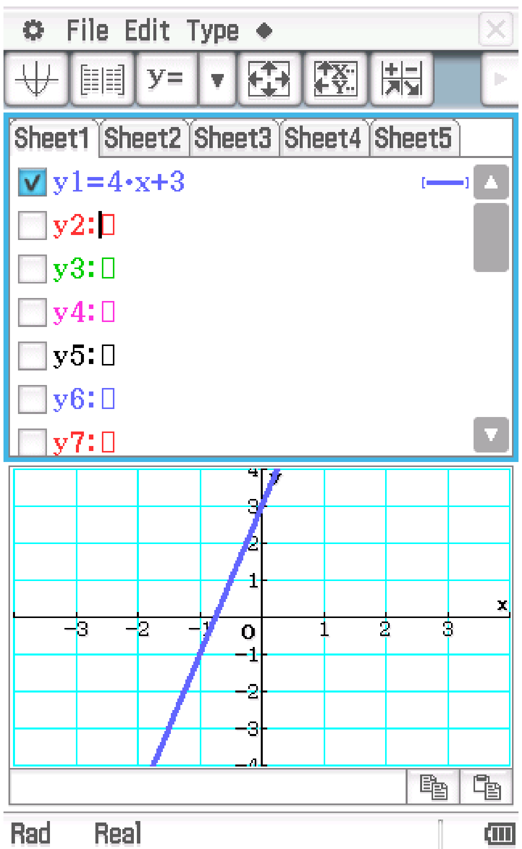
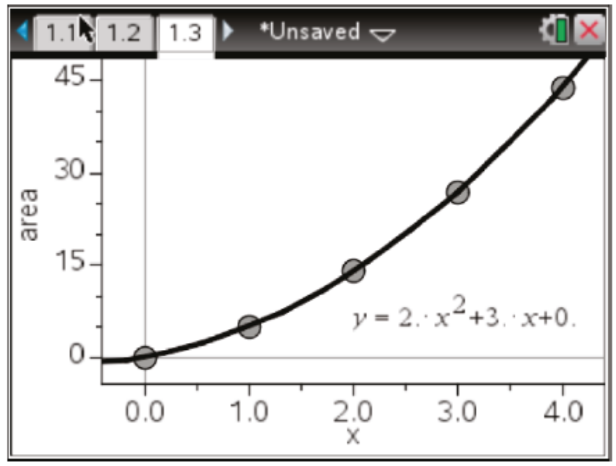
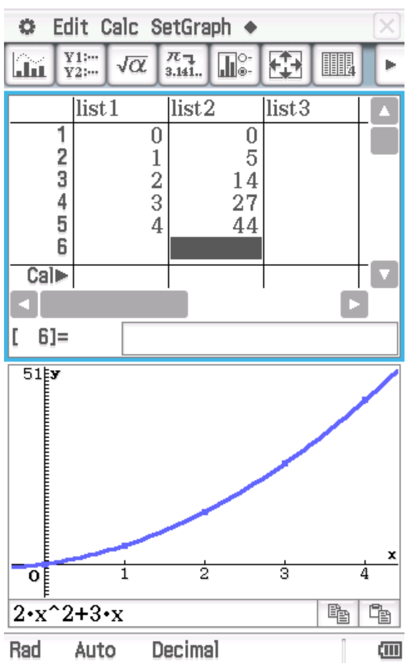


b TI-Nspire CAS

Interval	Area
[0, 0]	0
[0, 1]	5
[0, 2]	14
[0, 3]	27
[0, 4]	44



ClassPad



c $A(x) = 2x^2 + 3x$

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4 **a** $\int_0^6 x^2 dx = \left[\frac{x^3}{3}\right]_0^6 = \frac{1}{3}\left(6^3 - 0^3\right) = 72$

b $\int_0^3 x^3 dx = \left[\frac{x^4}{4}\right]_0^3 = \frac{1}{4}\left(3^4 - 0^4\right) = 20.25$

c $\int_0^2 x^5 dx = \left[\frac{x^6}{6}\right]_0^2 = \frac{1}{6}\left(2^6 - 0\right) = 10\frac{2}{3}$

d $\int_0^4 x^7 dx = \left[\frac{x^8}{8}\right]_0^4 = \frac{1}{8}\left(4^8 - 0\right) = 8192$

e $\int_0^5 x^4 dx = \left[\frac{x^5}{5}\right]_0^5 = \frac{1}{5}\left(5^5 - 0\right) = 625$

5 **a** $\int_1^3 x^2 dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{1}{3}\left(3^3 - 1^3\right) = 8\frac{2}{3}$

b $\int_2^8 x\, dx = \left[\frac{x^2}{2}\right]_2^8 = \frac{1}{2}\left(8^2 - 2^2\right) = 30$

c $\int_3^5 x^4 dx = \left[\frac{x^5}{5}\right]_3^5 = \frac{1}{5}\left(5^5 - 3^5\right) = 576.4$

d $\int_3^4 x^3 dx = \left[\frac{x^4}{4}\right]_3^4 = \frac{1}{4}\left(4^4 - 3^4\right) = 43.75$

e $\int_1^6 x^2 dx = \left[\frac{x^3}{3}\right]_1^6 = \frac{1}{3}\left(6^3 - 1^3\right) = 71\frac{2}{3}$

6 **a** $\int_2^6 x^5 dx = \left[\frac{x^6}{6}\right]_2^6 = \frac{1}{6}\left(6^6 - 2^6\right) = 7765.33$

b $\int_1^4 x^9 dx = \left[\frac{x^{10}}{10}\right]_1^4 = \frac{1}{10}\left(4^{10} - 1^{10}\right) = 104\ 857.5$

c $\int_4^6 x\, dx = \left[\frac{x^2}{2}\right]_4^6 = \frac{1}{2}\left(6^2 - 4^2\right) = 10$

d $\int_1^2 x^5 dx = \left[\frac{x^6}{6}\right]_1^2 = \frac{1}{6}\left(2^6 - 1^6\right) = 10.5$

e $\int_2^3 x^3 dx = \left[\frac{x^4}{4}\right]_2^3 = \frac{1}{4}\left(3^4 - 2^4\right) = 16.25$

f $\int_1^4 x^4 dx = \left[\frac{x^5}{5}\right]_1^4 = \frac{1}{5}\left(4^5 - 1^5\right) = 204.6$

g $\int_2^5 x\, dx = \left[\frac{x^2}{2}\right]_2^5 = \frac{1}{2}\left(5^2 - 2^2\right) = 10.5$

h $\int_3^5 x^7 dx = \left[\frac{x^8}{8}\right]_3^3 = \frac{1}{8}\left(5^8 - 3^8\right) = 48\ 008$

i $\int_1^2 x^9 dx = \left[\frac{x^{10}}{10}\right]_1^2 = \frac{1}{10}\left(2^{10} - 1^{10}\right) = 102.3$

j $\int_3^6 x^5 dx = \left[\frac{x^6}{6}\right]_3^6 = \frac{1}{6}\left(6^6 - 3^6\right) = 7654.5$

Reasoning and communication

7 **a** **i** $S = t^2$ m/s, $S_5 = 25$ m/s

ii $S_{10} = 100$ m/s

b Distance travelled = $\int_0^5 t^2 dt = \left[\frac{t^3}{3}\right]_0^5 = \frac{1}{3}\left(5^3 - 0\right) = 41.67$ m

c Distance travelled = $\int_5^{10} t^2 dt = \left[\frac{t^3}{3}\right]_5^{10} = \frac{1}{3}\left(10^3 - 5^3\right) = 291.67$ m

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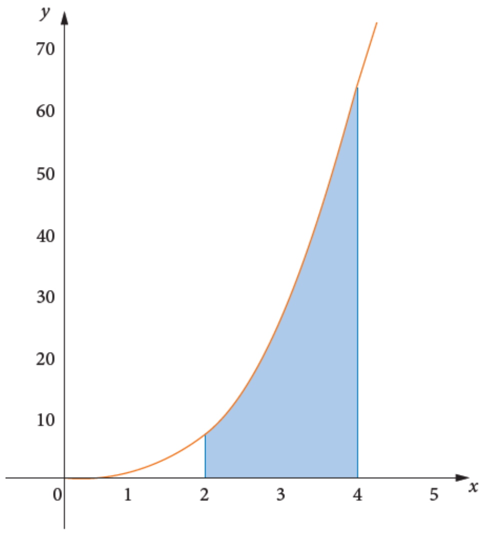
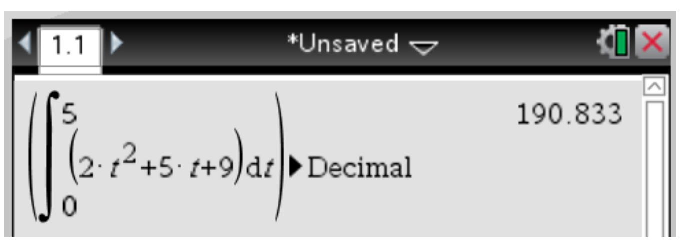
8 **a**



b $\int_2^4 x^3 dx$

c $\int_2^4 x^3 dx = \left[\frac{x^4}{4}\right]_2^4 = \frac{1}{4}\left(4^4 - 2^4\right) = 60$

9 **a** **TI-Nspire CAS**



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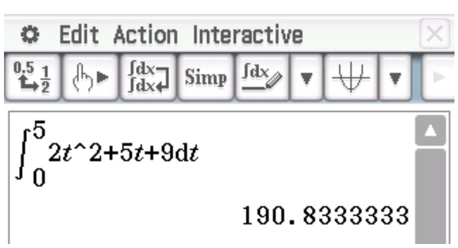

$\int_0^5 (2t^2 + 5t + 9)dt = 190.83$

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b Distance travelled $\int_0^5 \left(2t^2 + 5t + 9\right) dt = 190.33$ m

c Distance travelled $\int_3^5 \left(2t^2 + 5t + 9\right) dt = 123.33$ m

10 **a** $a = t^3$ m/s^2, $a_2 = 8$ m/s^2

b $v = \int_0^2 t^3 \, dt = 4$ m/s

c $v = \int_2^4 t^3 \, dt = 60$ m/s

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Exercise 4.06 Calculation of definite integrals

Concepts and techniques

1 **a** $\int_1^3 4x\,dx = \left[2x^2\right]_1^3 = 16$

b $\int_0^2 7x^6\,dx = \left[x^7\right]_0^2 = 128$

c $\int_1^2 4x^3\,dx = \left[x^4\right]_1^2 = 15$

d $\int_2^3 (2x-1)dx = \left[x^2 - x\right]_2^3 = (9-3)-(4-2) = 4$

e $\int_0^4 (x+2)dx = \left[\frac{x^2}{2} + 2x\right]_0^4 = (8+8)-(0) = 16$

f $\int_1^5 (6x-5)dx = \left[3x^2 - 5x\right]_1^5 = (75-25)-(3-5) = 52$

g $\int_0^1 (x^3 - 3x^2 + 1)dx = \left[\frac{x^4}{4} - x^3 + x\right]_0^1 = \left(\frac{1}{4} - 1 + 1\right) - 0 = \frac{1}{4}$

h $\int_0^3 (x^2 - x - 2)dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_0^3 = \left(9 - \frac{9}{2} - 6\right) - 0 = -1.5$

i $\int_1^2 (8x^3 - 5)dx = \left[2x^4 - 5x\right]_1^2 = (32-10)-(2-5) = 25$

j $\int_0^1 (x^4 - x^2 + 1)dx = \left[\frac{x^5}{5} - \frac{x^3}{3} + x\right]_0^1 = \left(\frac{1}{5} - \frac{1}{3} + 1\right) - 0 = \frac{13}{15}$

2 **a** $\int_0^2 \frac{x^2}{2}dx = \frac{1}{6}\left[x^3\right]_0^2 = \frac{1}{6}(8-0) = 1\frac{1}{3}$

b $\int_{-1}^1 (3x^2 + 4x)dx = \left[x^3 + 2x^2\right]_{-1}^1 = (1+2)-(-1+2) = 2$

c $\int_{-1}^2 (x^2 + 1)dx = \left[\frac{x^3}{3} + x\right]_{-1}^2 = \left(\frac{8}{3} + 2\right) - \left(\frac{-1}{3} - 1\right) = 6$

d $\int_{-2}^3 (4x^3 - 3)dx = \left[x^4 - 3x\right]_{-2}^3 = (81-9)-(16+6) = 50$

e $\int_{-1}^0 (x^2 + 3x + 5)dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} + 5x\right]_{-1}^0 = (0) - \left(\frac{-1}{3} + \frac{3}{2} - 5\right) = 3\frac{5}{6}$

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3 **a** $\int_0^4 e^x dx = \left[e^x\right]_0^4 = e^4 - 1$

b $\int_1^3 5e^x dx = 5\left[e^x\right]_1^3 = 5\left(e^3 - e\right) = 5e\left(e^2 - 1\right)$

c $\int_0^2 (2e^x + x)dx = \left[2e^x + \frac{x^2}{2}\right]_0^2 = \left(2e^2 + 2\right) - (2) = 2e^2$

d $\int_1^5 (e^x - 1)dx = \left[e^x - x\right]_1^5 = \left(e^5 - 5\right) - (e - 1) = e^5 - e - 4$

e $\int_2^4 (x^3 - e^x)dx = \left[\frac{x^4}{4} - e^x\right]_2^4 = \left(64 - e^4\right) - \left(4 - e^2\right) = 60 - e^4 + e^2$

4 **a** $\int_0^{\frac{\pi}{4}} \cos(x)dx = \left[\sin(x)\right]_0^{\frac{\pi}{4}} = \sin\left(\frac{\pi}{4}\right) - \sin(0) = \frac{1}{\sqrt{2}}$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin(x)dx = -\left[\cos(x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\left[\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}\right)\right] = -\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3} - 1}{2}$

c $\int_0^{\pi} 3\sin(x)dx = -3\left[\cos(x)\right]_0^{\pi} = -3\left[\cos(\pi) - \cos(0)\right] = -3(-1 - 1) = 6$

d $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos(x)dx = 2\left[\sin(x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 2\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)\right] = 2\left(1 - \frac{1}{\sqrt{2}}\right)$

e $\int_0^{\frac{\pi}{2}} 7\sin(x)dx = -7\left[\cos(x)\right]_0^{\frac{\pi}{2}} = -7\left[\cos\left(\frac{\pi}{2}\right) - \cos(0)\right] = -7(0 - 1) = 7$

5 **a** $\int_0^{\pi} \left[x + \sin(x)\right]dx = \left[\frac{x^2}{2} - \cos(x)\right]_0^{\pi} = \left[\frac{\pi^2}{2} - (-1)\right] - (0 - 1) = \frac{\pi^2}{2} + 2$

b $\int_0^{\frac{\pi}{4}} \left[\cos(x) - \sin(x)\right]dx = \left[\sin(x) - \cos(x)\right]_0^{\frac{\pi}{4}}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 - 1) = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$

c $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[\cos(x) + 1\right]dx = \left[\sin(x) + x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) - \left(\frac{1}{2} + \frac{\pi}{6}\right) = \frac{3\sqrt{3} + \pi - 3}{6}$

d $\int_0^{\frac{\pi}{3}} \left[2\sin(x) + 3\cos(x)\right]dx = \left[-2\cos(x) + 3\sin(x)\right]_0^{\frac{\pi}{3}} x$

e $\int_1^3 \left[\sin(x) + 3x^2\right]dx = \left[-\cos(x) + x^3\right]_1^3$

$= \left[-\cos(3) + 27\right] - \left[-\cos(1) + 1\right] = 27.53$

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6 **a** $\int_0^{\pi} 3\cos(x)\,dx = 3\left[\sin(x)\right]_0^{\pi} = 3\left[\sin(\pi)-\sin(0)\right] = 0$

b $\int_{\pi}^{\frac{4\pi}{3}} \cos(x)\,dx = \left[\sin(x)\right]_{\pi}^{\frac{4\pi}{3}} = \left[\sin\left(\frac{4\pi}{3}\right)-\sin(\pi)\right] = -\frac{\sqrt{3}}{2}$

c $\int_{\frac{2\pi}{3}}^{\frac{3\pi}{2}} 3\sin(x)\,dx = -3\left[\cos(x)\right]_{\frac{2\pi}{3}}^{\frac{3\pi}{2}} = -3\left[\cos\left(\frac{3\pi}{2}\right)-\cos\left(\frac{2\pi}{3}\right)\right] = -3\left[0-(-0.5)\right] = -1.5$

d $\int_{\pi}^{\frac{5\pi}{4}} \sqrt{2}\cos(x)\,dx = \sqrt{2}\left[\sin(x)\right]_{\pi}^{\frac{5\pi}{4}} = \sqrt{2}\left[\sin\left(\frac{5\pi}{4}\right)-\sin(\pi)\right] = \sqrt{2}\left(-\frac{1}{\sqrt{2}}-0\right) = -1$

e $\int_{\pi}^{\frac{11\pi}{6}} 2\sin(x)\,dx = -2\left[\cos(x)\right]_{\pi}^{\frac{11\pi}{6}}$

$$= -2\left[\cos\left(\frac{11\pi}{6}\right)-\cos(\pi)\right] = -2\left[\frac{\sqrt{3}}{2}-(-1)\right] = -\sqrt{3}-2$$

Reasoning and communication

7 **a** $\int_0^3 (2x-1)dx + \int_3^5 (2x-1)dx = \int_0^5 (2x-1)dx = \left[x^2-x\right]_0^5 = 20$

b $\int_0^4 e^x dx + \int_0^4 x\,dx = \int_0^4 \left(e^x + x\right)dx = \left[e^x + \frac{x^2}{2}\right]_0^4 = \left(e^4+8\right)-(e^0+0) = e^4+7$

c $\int_0^{\frac{\pi}{6}} \cos(x)dx - \int_0^{\frac{\pi}{6}} 2\sin(x)dx = \int_0^{\frac{\pi}{6}} \left[\cos(x)-2\sin(x)\right]dx$

$$= \left[\sin(x)+2\cos(x)\right]_0^{\frac{\pi}{6}}$$
$$= \left[\sin\left(\frac{\pi}{6}\right)+2\cos\left(\frac{\pi}{6}\right)\right]-\left[\sin(0)+2\cos(0)\right]$$
$$= \left(\frac{1}{2}+\sqrt{3}\right)-(2)$$
$$= \sqrt{3}-1.5$$

8 **a** $\frac{d}{dx}\left[\tan(x)\right] = \frac{1}{\cos^2(x)}$

b $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^2(x)}dx = \left[\tan(x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \tan\left(\frac{\pi}{3}\right)-\tan\left(\frac{\pi}{4}\right) = \sqrt{3}-1$

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9 **a** $\frac{d}{dx}(e^{4x}) = 4e^{4x}$

b $\int_0^3 4e^{4x}dx = \left[e^{4x}\right]_0^3 = e^{12} - 1$

c $\int_0^3 e^{4x}dx = \frac{1}{4}\times\left[e^{4x}\right]_0^3 = \frac{e^{12}-1}{4}$

10 $v = \frac{dx}{dt} = 3t^2 + 2t - 5$ cm/s

a $v_0 = -5$ cm/s

b $x = \int\left(3t^2 + 2t - 5\right)dt = t^3 + t^2 - 5t + c$

At $t = 2$, $x = 3 \Rightarrow 3 = 8 + 4 - 10 + c$, so $c = 1$

$\therefore \quad x = t^3 + t^2 - 5t + 1$

c $x_5 = 125 + 25 - 25 + 1$

$x_5 = 126$ cm

d $v = 3t^2 + 2t - 5$ cm/s

$a = 6t + 2$ cm/s^2

$a_3 = 20$ cm/s^2

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Exercise 4.07 Areas under curves

Concepts and techniques

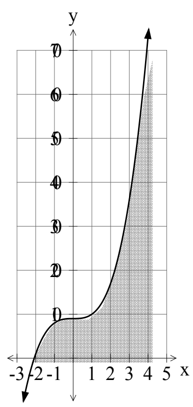
1 $\int_6^9 (4x+1)\,dx = \left[2x^2 + x\right]_6^9 = 171 - 78 = 93$

2 $\int_4^7 x^2\,dx = \left[\frac{x^3}{3}\right]_4^7 = \frac{1}{3}(279) = 93$

3 $\int_1^5 x^3\,dx = \left[\frac{x^4}{4}\right]_1^5 = \frac{1}{4}(624) = 156$

4 $\int_2^5 (x^2+3)\,dx = \left[\frac{x^3}{3} + 3x\right]_2^5 = \left(\frac{125}{3} + 15\right) - \left(\frac{8}{3} + 6\right) = 48$

5 The area is all above the x-axis



$$\int_{-2}^4 (x^3+9)\,dx = \left[\frac{x^4}{4} + 9x\right]_{-2}^4 = \left(\frac{256}{4} + 36\right) - \left(\frac{16}{4} - 18\right) = 114$$

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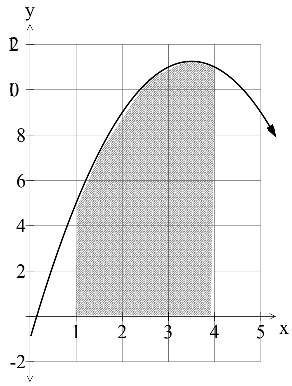
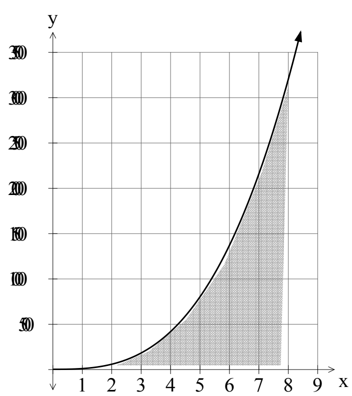
6 $y = 7x - x^2 - 1$



$$\int_1^4 \left(7x - x^2 - 1\right) dx = \left[\frac{7x^2}{2} - \frac{x^3}{3} - x\right]_1^4 = \left(56 - \frac{64}{3} - 4\right) - \left(\frac{7}{2} - \frac{1}{3} - 1\right) = \left(30\frac{2}{3}\right) - \left(2\frac{1}{6}\right)$$

$= 28.5$ units^2

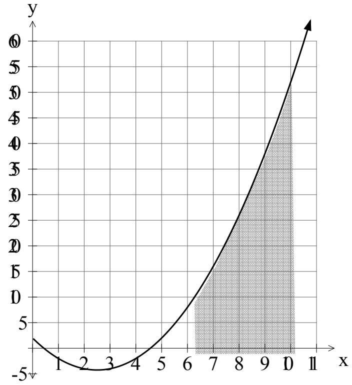
7 $y = 6x^3 + 2x^2 + 3$



$$\int_2^8 \left(6x^3 + 2x^2 + 3\right) dx = \left[\frac{3x^4}{2} + \frac{2x^3}{3} + 3x\right]_2^8 = \left(6144 + 341\frac{1}{3} + 24\right) - \left(24 + \frac{16}{3} + 6\right)$$

$= 6474$ units^2

8 $y = x^2 - 5x + 2$

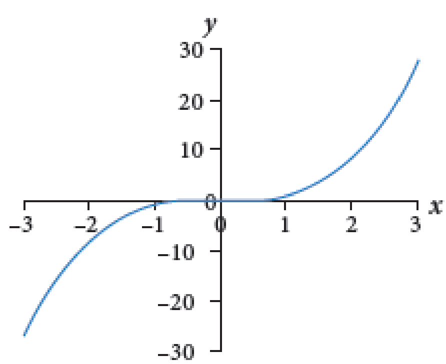


$$\int_6^{10}\left(x^2 - 5x + 2\right)dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 2x\right]_6^{10} = \left(\frac{1000}{3} - 250 + 20\right) - \left(\frac{216}{3} - 90 + 12\right)$$

$$= 109\frac{1}{3} \text{ units}^2$$

Reasoning and communication

9 **a**



b $\int_{-2}^{0} x^3 dx = \left[\frac{x^4}{4}\right]_{-2}^{0} = (0) - (4) = -4$

c $\int_0^2 x^3 dx \left[\frac{x^4}{4}\right]_0^2 = (4) - (0) = 4$

d $\int_{-2}^{2} x^3 dx = \left[\frac{x^4}{4}\right]_{-2}^{2} = (4) - (4) = 0$

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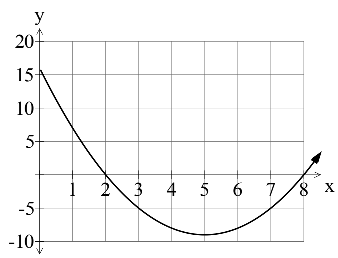
e Area = 8 units^2

f As the function is below the x-axis, the y values are negative so the 'area' in that part is given as negative. The area is $|-4| + 4 = 8$

10 **a** $f(x) = x^2 - 10x + 16$



Negative.

b $\int_3^7 (x^2 - 10x + 16)dx = \left[\frac{x^3}{3} - 5x^2 + 16x\right]_3^7$

$$= \left(114\frac{1}{3} - 245 + 112\right) - (9 - 45 + 48) = -30\frac{2}{3}$$

c Area = 30.67 units^2

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Chapter 4 Review

Multiple choice

1 B $\Delta x = 0.5$ and using points 1.5, 2, 2.5, 3 gives $0.5(1.5^2 + 2^2 + 2.5^2 + 3^2)$

2 D $\Delta x = 1$ and using points 0.5, 1.5, 2.5, 3.5, 4.5

gives $1 \times [(0.5^3 + 1) + (1.5^3 + 1) + (2.5^3 + 1) + (3.5^3 + 1) + (4.5^3 + 1)]$

3 D $\int_2^4 (3x^3 - 5x^2 + 4x + 1)dx - \int_2^4 (x^3 + x^2 - 5x - 3)dx$

$= \int_2^4 (3x^3 - 5x^2 + 4x + 1) - (x^3 + x^2 - 5x - 3)dx$

$= \int_2^4 (2x^3 - 6x^2 + 9x + 4)dx$

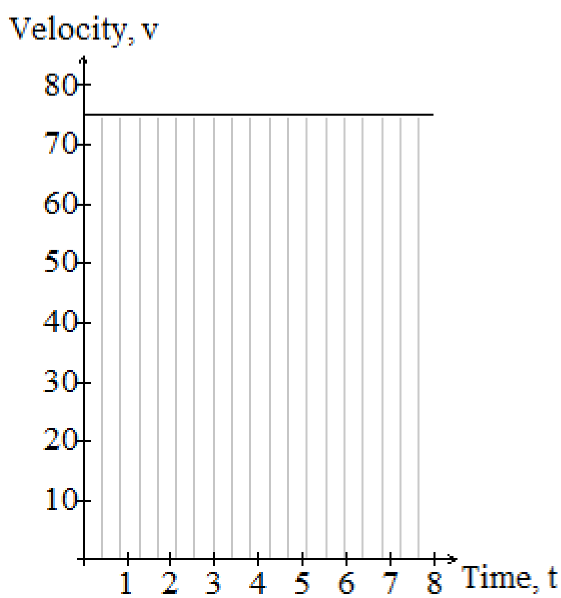
4 D $\int_{-2}^{2} (12x^2 - 6x + 5)\,dx = \left[4x^3 - 3x^2 + 5x\right]_{-2}^{2} = (32 - 12 + 10) - (-32 - 12 - 10) = 84$

5 C The area $= \int_2^4 \left(x^2 - 1\right)dx = \left[\frac{x^3}{3} - x\right]_2^4 = \left(\frac{64}{3} - 4\right) - \left(\frac{8}{3} - 2\right) = 16\frac{2}{3}$

Short answer

6 **a** Distance travelled = 75 × 8 = 600 km

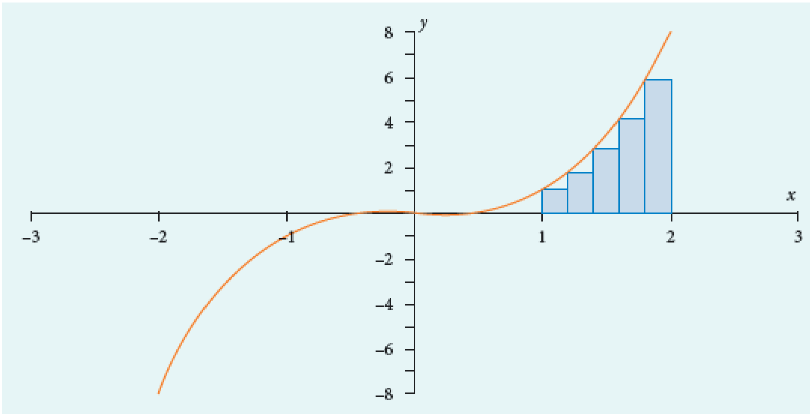
b



7 The approximate distance travelled by a particle between 1 and 5 seconds

$= 3 \times (5 - 1) = 12$ m

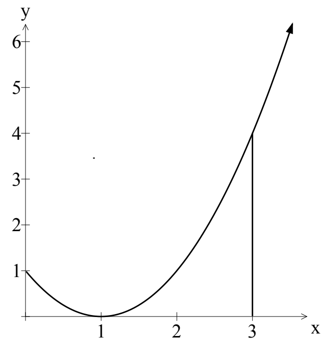
8 $y = x^3$



Area $\approx 0.2(1^3 + 1.2^3 + 1.4^3 + 1.6^3 + 1.8^3)$

$= 3.08$ units^2

9 $y = x^2 - 2x + 1$



a **i** Area $\approx 0.5[f(1) + f(1.5) + f(2) + f(2.5)]$

$= 0.5[0 + 0.25 + 1 + 2.25]$

$= 1.75$

ii Area $\approx 0.5[f(1.5) + f(2) + f(2.5) + f(3)]$

$= 0.5[0.25 + 1 + 2.25 + 4]$

$= 3.75$

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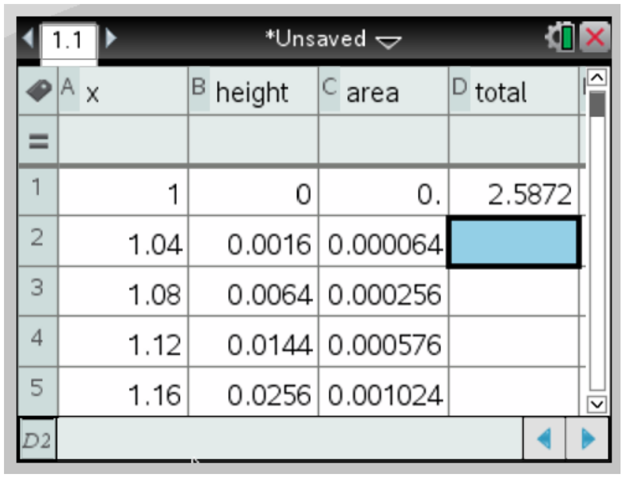
b

TI-Nspire CAS



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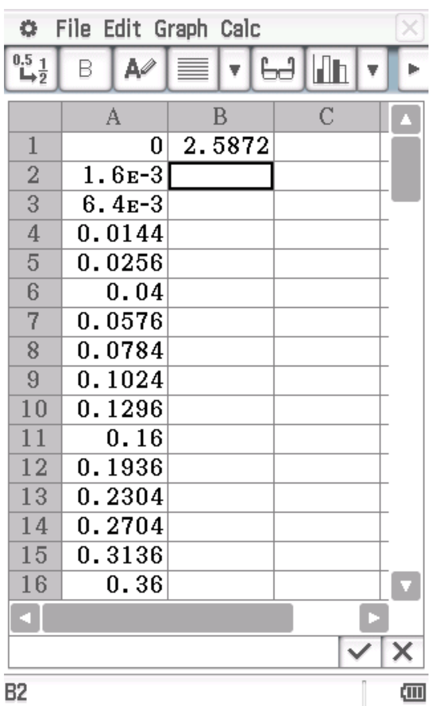


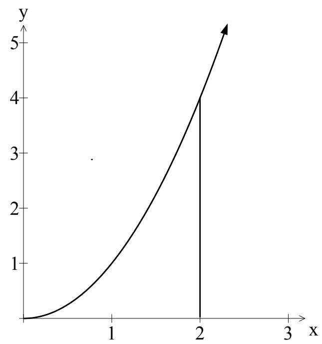
$\Delta x = 2 \div 50 = 0.04$, so width = 0.04

Counter		$y = x^2 - 2x + 1$
1	1	0
2	1.04	0.0016
3	1.08	0.0064
4	1.12	0.0144
5	1.16	0.0256
6	1.2	0.04
7	1.24	0.0576
8	1.28	0.0784
9	1.32	0.1024
10	1.36	0.1296
11	1.4	0.16
12	1.44	0.1936
13	1.48	0.2304
14	1.52	0.2704
15	1.56	0.3136
16	1.6	0.36
17	1.64	0.4096
18	1.68	0.4624
19	1.72	0.5184
20	1.76	0.5776
21	1.8	0.64
22	1.84	0.7056
23	1.88	0.7744
24	1.92	0.8464
25	1.96	0.9216

Counter		$y = x^2 - 2x + 1$
26	2	1
27	2.04	1.0816
28	2.08	1.1664
29	2.12	1.2544
30	2.16	1.3456
31	2.2	1.44
32	2.24	1.5376
33	2.28	1.6384
34	2.32	1.7424
35	2.36	1.8496
36	2.4	1.96
37	2.44	2.0736
38	2.48	2.1904
39	2.52	2.3104
40	2.56	2.4336
41	2.6	2.56
42	2.64	2.6896
43	2.68	2.8224
44	2.72	2.9584
45	2.76	3.0976
46	2.8	3.24
47	2.84	3.3856
48	2.88	3.5344
49	2.92	3.6864
50	2.96	3.8416
	sum =	**64.68**

Area = 0.04 × 64.68 = 2.5872 units^2

10 $y = x^2$



a The approximate area under the curve $y = x^2$ between $x = 0$ and $x = 2$ using

$= 0.25[f(0) + f(0.25) + f(0.5) + \ldots + f(1.75)]$

$= 2.1875$

b The approximate area under the curve $y = x^2$ between $x = 0$ and $x = 2$ using

$= 0.25[f(0.25) + f(0.5) + \ldots + f(2)]$

$= 3.1875$

c The approximate area under the curve $y = x^2$ between $x = 0$ and $x = 2$ using

$= 0.25[\ f(0.125) + f(0.375) + f(0.625) + \ldots.. + f(1.875)\ ]$

$= 2.656\ 25$

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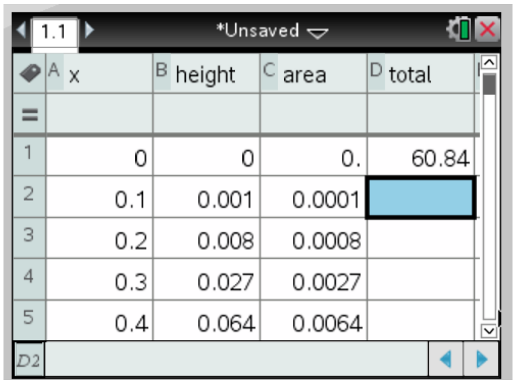
11 TI-Nspire CAS



	A x	B height	C area	D total
=				
1	0	0	0.	60.84
2	0.1	0.001	0.0001	
3	0.2	0.008	0.0008	
4	0.3	0.027	0.0027	
5	0.4	0.064	0.0064	

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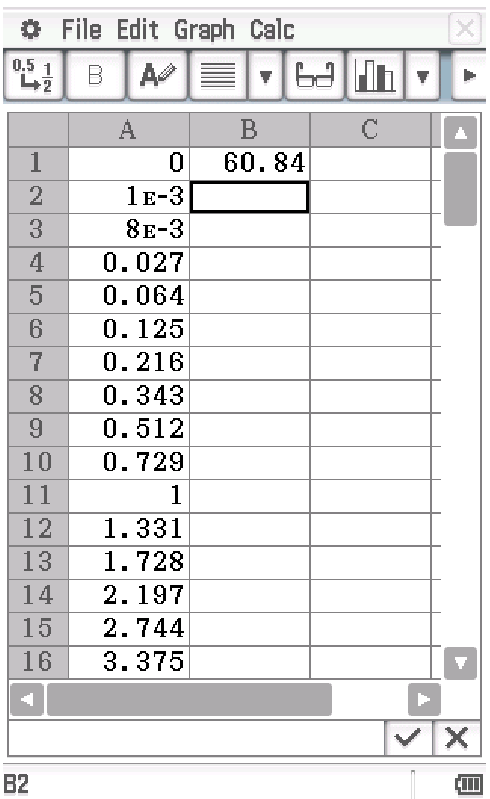


	A	B	C
1	0	60.84	
2	1E-3		
3	8E-3		
4	0.027		
5	0.064		
6	0.125		
7	0.216		
8	0.343		
9	0.512		
10	0.729		
11	1		
12	1.331		
13	1.728		
14	2.197		
15	2.744		
16	3.375		

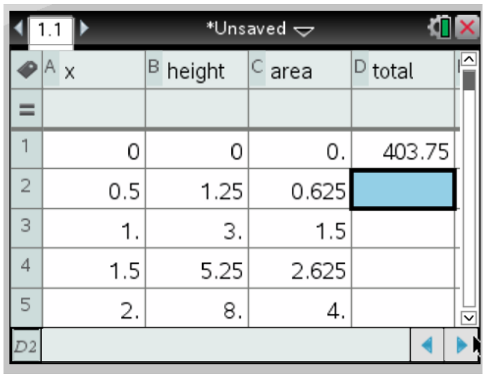
The approximate area under the curve $y = x^3$ between $x = 0$ and $x = 4$ using

40 left rectangles, using $\Delta x = 0.1$

$\approx 0.1 \times [\, f(0) + f(0.1) + f(0.2) + \ldots.. + f(3.9)\,]$

$= 60.84$ units^2

12 TI-Nspire CAS



	A x	B height	C area	D total
=				
1	0	0	0.	403.75
2	0.5	1.25	0.625	
3	1.	3.	1.5	
4	1.5	5.25	2.625	
5	2.	8.	4.	

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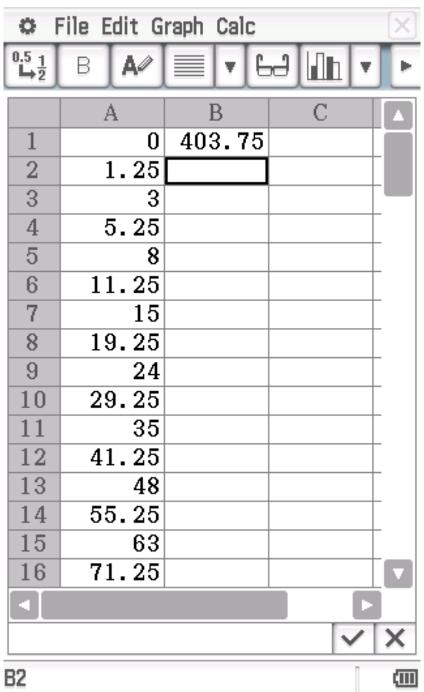

	A	B	C
1	0	403.75	
2	1.25		
3	3		
4	5.25		
5	8		
6	11.25		
7	15		
8	19.25		
9	24		
10	29.25		
11	35		
12	41.25		
13	48		
14	55.25		
15	63		
16	71.25		

$$\int_0^{10} (x^2 + 2x)dx \approx 0.5[f(0) + f(0.5) + f(1) + ... + f(9.5)] = 403.75$$

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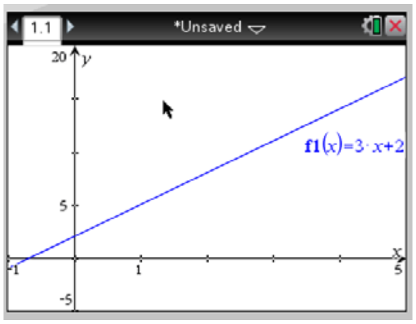
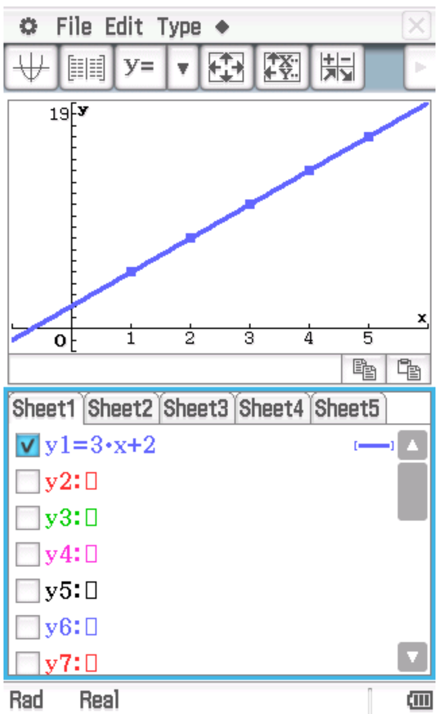
13 TI-Nspire CAS



ClassPad



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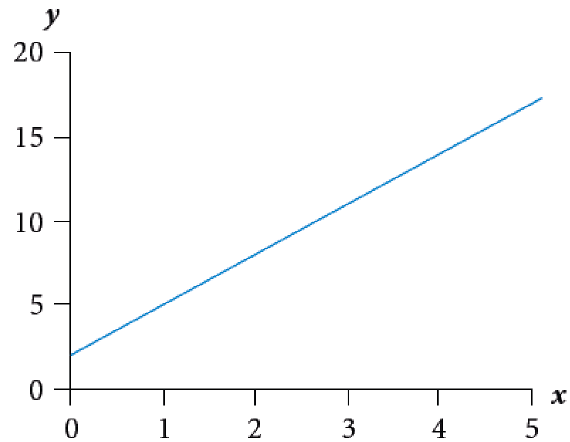
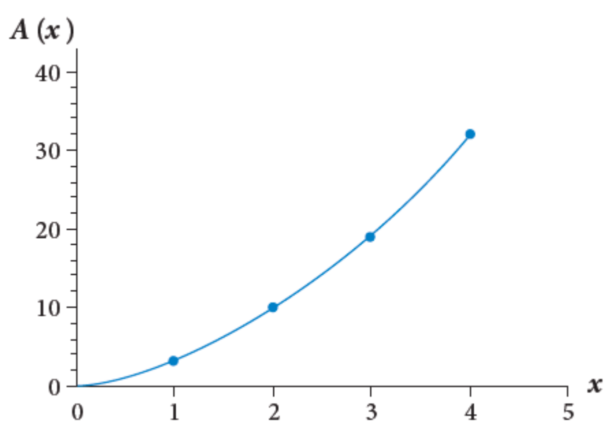
a $y = 3x + 2$ for $x = 0$ to 5

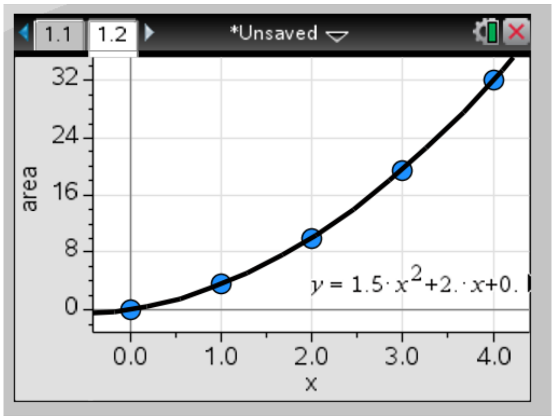


b

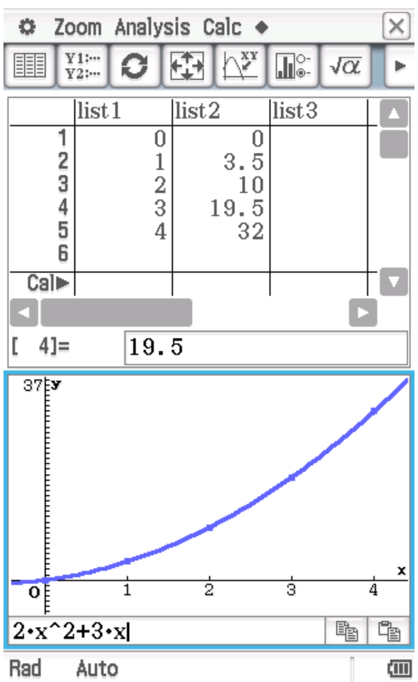
Interval	Area
[0, 0]	0
[0, 1]	3.5
[0, 2]	10
[0, 3]	19.5
[0, 4]	32



c TI-Nspire CAS



ClassPad



$A(x) = 1.5x^2 + 2x$

14 **a** **i** $\int_1^2 (2x^2+1)dx \approx 0.5[f(1.5) + f(2)] = 7.25$

ii $\int_2^4 (2x^2+1)dx \approx 0.5[f(2.5) + f(3) + ... + f(4)] = 45.5$

iii $\int_1^4 (2x^2+1)dx \approx 0.5[f(1.5) + f(2) + ... + f(4)] = 52.75$

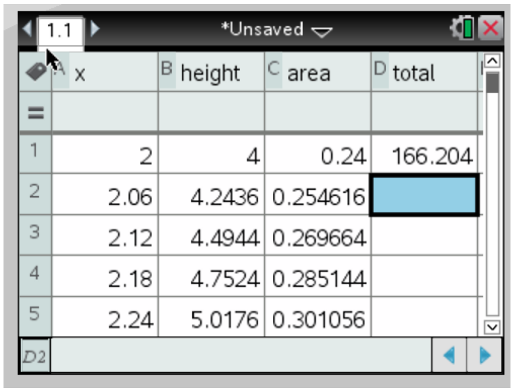
b Show that $\int_1^4 (2x^2+1)dx = \int_1^2 (2x^2+1)dx + \int_2^4 (2x^2+1)dx$

$\int_1^4 (2x^2+1)dx = 52.75$

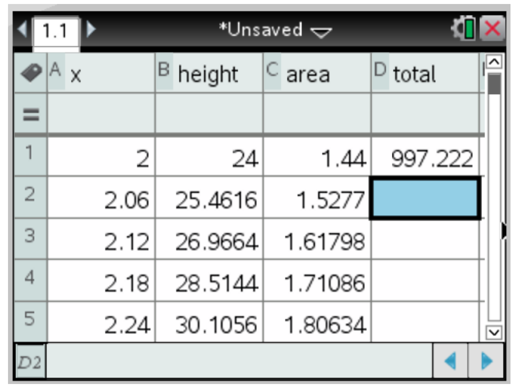
$\int_1^2 (2x^2+1)dx + \int_2^4 (2x^2+1)dx = 7.25 + 45.5 = 52.75$

$\therefore \int_1^4 (2x^2+1)dx = \int_1^2 (2x^2+1)dx + \int_2^4 (2x^2+1)dx$

15 TI-Nspire CAS

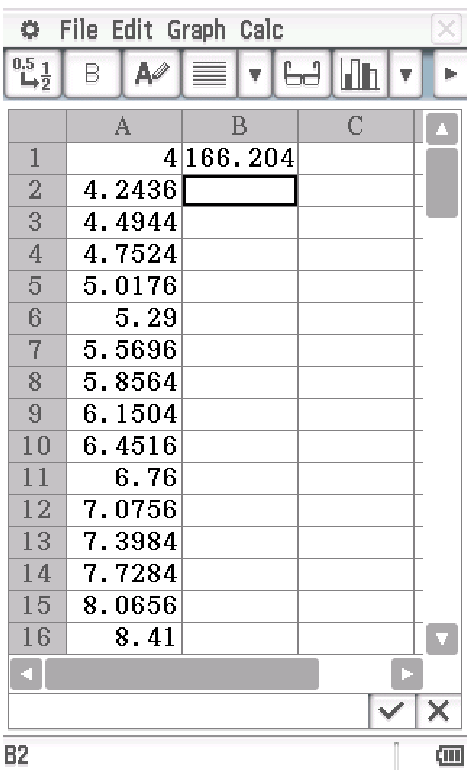


	A x	B height	C area	D total
=				
1	2	4	0.24	166.204
2	2.06	4.2436	0.254616	
3	2.12	4.4944	0.269664	
4	2.18	4.7524	0.285144	
5	2.24	5.0176	0.301056	

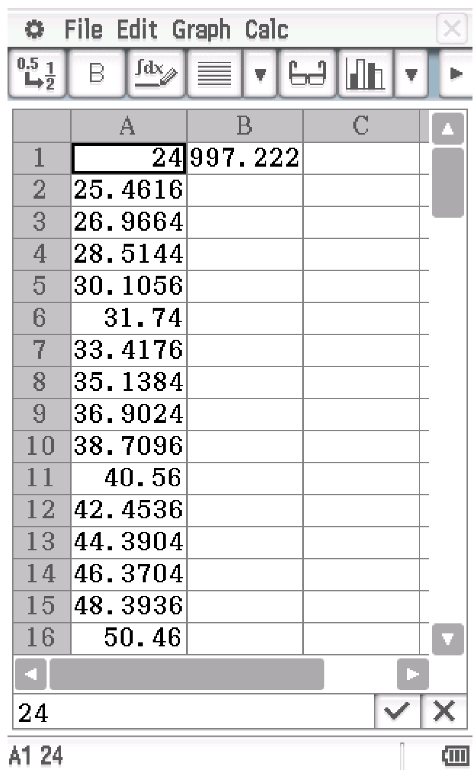


	A x	B height	C area	D total
=				
1	2	24	1.44	997.222
2	2.06	25.4616	1.5277	
3	2.12	26.9664	1.61798	
4	2.18	28.5144	1.71086	
5	2.24	30.1056	1.80634	

ClassPad



	A	B	C
1	4	166.204	
2	4.2436		
3	4.4944		
4	4.7524		
5	5.0176		
6	5.29		
7	5.5696		
8	5.8564		
9	6.1504		
10	6.4516		
11	6.76		
12	7.0756		
13	7.3984		
14	7.7284		
15	8.0656		
16	8.41		



	A	B	C
1	24	997.222	
2	25.4616		
3	26.9664		
4	28.5144		
5	30.1056		
6	31.74		
7	33.4176		
8	35.1384		
9	36.9024		
10	38.7096		
11	40.56		
12	42.4536		
13	44.3904		
14	46.3704		
15	48.3936		
16	50.46		

a **i** Use $\Delta x = 6 \div 100 = 0.06$ and use points 2, 2.06, etc.

$$\int_2^8 x^2 dx = 0.06 \times \left(2^2 + 2.06^2 + \cdots + 7.94^2\right) = 166.204 \text{ units}^2$$

ii Use $\Delta x = 6 \div 100 = 0.06$ and use points 2, 2.06, etc.

$$\int_2^8 6x^2 dx = 0.06 \times 6\left(2^2 + 2.06^2 + \cdots + 7.94^2\right) = 997.222 \text{ units}^2$$

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b $\int_2^8 x^2 dx = 166.204 \text{ units}^2$

$\int_2^8 6x^2 dx = 997.222 \text{ units}^2$

$6 \times 166.204 = 997.222$

$\therefore \quad \int_2^8 6x^2 dx = 6\int_2^8 x^2 dx$

16 **a** **i** $\int_1^2 x^3 dx \approx 0.25 \times \left(1^3 + 1.25^3 + 1.5^3 + 1.75^3\right) = 2.92$

ii $\int_1^2 2x\, dx \approx 0.25 \times \left(2 \times 1 + 2 \times 1.25 + 2 \times 1.5 + 2 \times 1.75\right) = 2.75$

iii $\int_1^2 (x^3 + 2x) dx$

$\approx 0.25 \times \left(1^3 + 2 \times 1 + 1.25^3 + 2 \times 1.25 + 1.5^3 + 2 \times 1.5 + 2^3 + 2 \times 1.75\right)$

$= 5.671875$

b $\int_1^2 (x^3 + 2x) dx \approx 5.67$

$\int_1^2 x^3 dx + \int_1^2 2x\, dx = 2.92 + 2.75 = 5.67$

$\therefore \quad \int_1^2 (x^3 + 2x) dx = \int_1^2 x^3 dx + \int_1^2 2x\, dx$

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17 **a** $\int_0^2 x^3 dx = \left[\frac{x^4}{4}\right]_0^2 = 4 - 0 = 4$

b $\int_1^3 x\, dx = \left[\frac{x^2}{2}\right]_1^3 = 4.5 - 0.5 = 4$

c $\int_0^3 (x^2 + 3x - 4)dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} - 4x\right]_0^3 = (9 + 13.5 - 12) - (0) = 10.5$

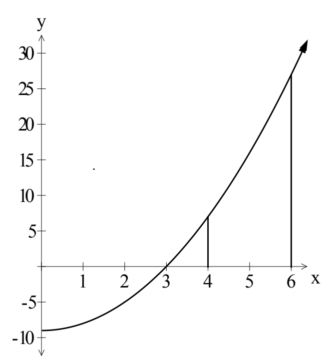
d $\int_1^2 (3x - 2)dx = \left[\frac{3x^2}{2} - 2x\right]_1^2 = (6 - 4) - (1.5 - 2) = 2.5$

18 $\int_0^7 3e^x dx = \left[3e^x\right]_0^7 = 3e^7 - 3e^0 = 3e^7 - 3$

19 **a** $\int_0^{\frac{\pi}{4}} \sin(x)\, dx = -\left[\cos(x)\right]_0^{\frac{\pi}{4}} = -\left[\cos\left(\frac{\pi}{4}\right) - \cos(0)\right] = 1 - \frac{1}{\sqrt{2}}$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos(x)\, dx = \left[\sin(x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left[\sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)\right] = \frac{\sqrt{3}}{2} - \frac{1}{2}$

20 **a**

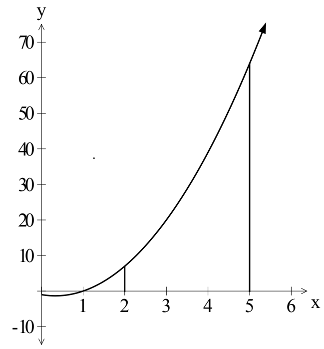


$$\int_4^6 \left(x^2 - 9\right) dx = \left[\frac{x^3}{3} - 9x\right]_4^6$$

$$= \left(\frac{216}{3} - 54\right) - \left(\frac{64}{3} - 36\right)$$

$$= 32\frac{2}{3} \text{ units}^2$$

b $f(x) = 3x^2 - 2x - 1$



$$\int_2^5 \left(3x^2 - 2x - 1\right) dx = \left[x^3 - x^2 - x\right]_2^5$$
$$= (125 - 25 - 5) - (8 - 4 - 2)$$
$$= 93 \text{ units}^2$$

Application

21 **a** $v = 3 \cos(t)$ cm/s

$v_1 = 3 \cos(1)$ cm/s = 1.62

b Distance travelled in the first second

$$= \int_0^1 3\cos(t) = 3\left[\sin(t)\right]_0^1 = 3\sin(1) - 3\sin(0) = 3\sin(1) = 2.52$$

c $\int_{0.6}^{0.9} 3\cos(t) = 3\left[\sin(t)\right]_{0.6}^{0.9} = 3\sin(0.9) - 3\sin(0.6) = 0.66$

22 **a** $\frac{d}{dx}\left(\frac{x}{e^x}\right) = \frac{1 \times e^x - e^x x}{e^{2x}} = \frac{1-x}{e^x}$

b $\int_0^1 \frac{1-x}{e^x} dx = \left[\frac{x}{e^x}\right]_0^1 = \frac{1}{e} - 0 = \frac{1}{e}$

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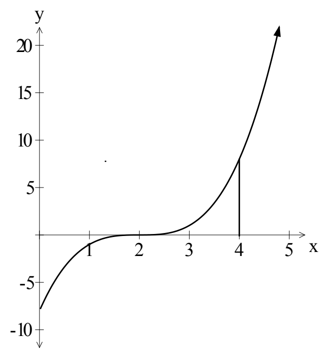
23



a Negative

b Positive

c $$\int_0^4 (x^3 - 6x^2 + 12x - 8)dx = \left[\frac{x^4}{4} - 2x^3 + 6x^2 - 8x\right]_0^4$$

$$= (64 - 128 + 96 - 32) - 0$$
$$= 0$$

d $$\int_0^2 (x^3 - 6x^2 + 12x - 8)dx = \left[\frac{x^4}{4} - 2x^3 + 6x^2 - 8x\right]_0^2$$

$$= (4 - 16 + 24 - 16) - 0$$
$$= -4$$

e The area between $f(x) = x^3 - 6x^2 + 12x - 8$ and the x-axis from $x = 0$ to $x = 4$ is 8 units^2 because it is anti-symmetrical about $x = 2$.

f Because the algebraic area is –4 + 4 = 0, but the physical area is 4 + 4 = 8, as it cannot be negative.

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